Joint Measurement of Target Angle and Angular Velocity Using Interferometric Radar with FM Waveforms

2020 IEEE Radar Conference

Jason Merlo* and Jeffrey A. Nanzer
Michigan State University, East Lansing, MI, USA
Motivation

Angle and Angular Rate Measurement

• **Angular rate is not directly measured** by conventional radar systems; instead a **locate and track method is typically used** to derive it

• Moreover common array angle estimations methods are complex and computationally expensive

• **Active correlation interferometry can be used to measure angle and angular rate** using direct frequency estimation, **analogously to range-Doppler measurement**
Applications

Angle and Angular Rate Measurement

- Human Computer Interfaces (HCI)
- Automotive Radar
- Airspace Monitoring
- Space-object Monitoring
Background

Correlation Interferometry

• Correlation interferometry can be used to measure angular rate of a target\(^{(1)}\)

• Phases across multiple apertures are summed; this produces an interference or “fringe” pattern

• Moving targets produce frequency proportional to angular rate after correlation

• Using *active* correlation interferometry both angular rate and angle can be estimated

---

The Interferometric Approach
Angular Rate Measurement

The Interferometric Approach

Correlator output:

\[ r_c(t) = A(\theta) \exp \left( j2\pi f_0 \tau_g \right) = A(\theta) \exp \left( j2\pi D_\lambda \sin \theta \right) \]

Angular rate measurement:

Using \( \omega = \frac{d\theta}{dt} \) \( \Rightarrow \) \( \theta = \omega t + \theta_0 \)

\[ f_\omega = \frac{1}{2\pi} \frac{d\phi_r(t)}{dt} = \omega D_\lambda \cos(\theta) \]

Finally, using \( \omega = \frac{v}{R} \)

\[ v_\theta \approx \frac{f_\omega R}{D_\lambda} \]

where \( \tau_g = \tau_{d2} - \tau_{d1} = \frac{D}{c} \sin \theta \)
Angle Measurement

The Interferometric Approach

• To measure absolute angle unambiguously, a modulated waveform is required

• Linear frequency modulation (LFM) will be used in this analysis

\[ \omega_t(t) = 2\pi(f_0 + Kt); \quad t \in [-\tau/2, \tau/2] \]

where \( K = \beta/\tau \) is the chirp rate
\( \beta \) is the chirp bandwidth
\( \tau \) is the chirp duration

\[ s_t(t) = A(\theta) \exp \left( j \int \omega_t(t) dt \right) = A(\theta) \exp \left[ j2\pi \left( f_0 t + \frac{K}{2} t^2 \right) \right] \]
Angle Measurement

The Interferometric Approach

Downconverted signal at $r_{dn}$:

$$r_{dn}(t) = r_n(t) \cdot s^*_t(t)$$

$$= A(\theta) \exp \left\{ j2\pi \left[ -f_0 \tau_{dn} + \frac{K}{2} \left( \tau_{dn}^2 - 2\tau_{dn}t \right) \right] \right\}$$

Correlation signal at $r_c$:

$$r_c(\tau_g, t) = r_1(t) \cdot r_2^*(t)$$

$$= A(\theta) \exp \left\{ j2\pi \left[ f_0 \tau_g + \frac{K}{2} \left( \tau_{d2}^2 - 2\tau_{d2}t \right) \right] \right\}$$
Angle Measurement

The Interferometric Approach

\[ r_c(\tau_g, t) = A(\theta) \exp\left\{ j2\pi \left[ f_0\tau_g + \frac{K}{2} \left[ (\tau_{d2}^2 - 2\tau_{d2}t) - (\tau_{d1}^2 - 2\tau_{d1}t) \right] \right] \right\} \]

\[ = A(\theta) \exp\left\{ j2\pi \left[ f_0\tau_g + \frac{K}{2} \left( \tau_{d2}^2 - \tau_{d1}^2 - 2\tau_g t \right) \right] \right\} \]

where \( \tau_g = \tau_{d2} - \tau_{d1} = \frac{D}{c} \sin \theta \)

Angle may be derived from the beat frequency:

\[ f_b(\theta, t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} \]

\[ = \frac{d}{dt} \left[ f_0\tau_g + \frac{K}{2} \left( \tau_{d2}^2 - \tau_{d1}^2 - 2\tau_g t \right) \right] \]
Angle Measurement

The Interferometric Approach

\[ f_b(\theta, t) = \frac{d}{dt} \left[ f_0 \tau_g + \frac{K}{2} \left( \tau_{d2}^2 - \tau_{d1}^2 - 2\tau_g t \right) \right] \]

Note that:

\[ \frac{d\tau_{dn}}{dt} = \frac{2v_{Rn}}{c} \quad \text{and} \quad \frac{d\theta}{dt} = \omega \quad \Rightarrow \quad \frac{d\tau_g}{dt} = \omega \frac{D}{c} \cos(\theta) \]

For a dynamic target \( v_{Rn} \neq 0 \), the beat frequency is:

\[ f_b(\theta, t) = \omega D \cos \theta - K \left[ \frac{D}{c} \left( \sin \theta + \omega t \cos \theta \right) - \frac{2}{c^2} \left( R_2 v_{R2} - R_1 v_{R1} \right) \right] \]

Angular Velocity \quad Angle \quad Intermodulation Terms
Angle Measurement

The Interferometric Approach

\[ f_b(\theta, t) \approx \omega D \lambda \cos \theta - K \left[ \frac{D}{c} (\sin \theta + \omega t \cos \theta) \right] \]

Integrating \( f_b \) over one chirp:

\[ f_b(\theta) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f_b(\theta, t) dt \approx \omega \tau D \lambda \cos \theta - \beta \frac{D}{c} \sin \theta \]
Angle Measurement

The Interferometric Approach

\[ f_b(\theta) \approx \omega \tau D \lambda \cos \theta - \beta \frac{D}{c} \sin \theta \]

Quasi-static approximation:

if \[ \sin \theta \gg \frac{f_0}{K} \cos \theta \]

then \[ \theta \approx \sin^{-1} \left( -\frac{c}{\beta D f_b} \right) \]
Angular Rate Measurement (LFM)

The Interferometric Approach

Angular rate is derived from the phase at the correlator, $\phi_c$

$$\phi_c(n) = 2\pi \left[ f_0 \tau_g(n) + \frac{K}{2} \left( \tau^2_{d2}(n) - \tau^2_{d1}(n) - 2\tau_g(n)t \right) \right]$$

Recall $\tau_g(n) = \frac{D}{c} \sin \theta(n)$ where $n$ is the number of pulses

$$f_\omega = \frac{1}{2\pi} \frac{d\phi_c}{dn} = \omega \frac{D}{c} \cos \theta \left( f_0 - \frac{\beta}{\tau} t \right); \quad t \in \left[ -\frac{\tau}{2}, \frac{\tau}{2} \right]$$

iff $f_0 \gg \frac{\beta}{2} \implies \omega \approx f_\omega \frac{1}{D_\lambda \cos \theta}$

or $v_\theta \approx f_\omega \frac{R}{D_\lambda}$ for small angles, and

$$\theta \approx \sin^{-1}\left( -\frac{c}{\beta D} f_\theta \right)$$
Validation of Theory
Simulated Linear Frequency Modulated Interferometer
Proposed Active LFM Interferometer

Validation of Theory - Linear Frequency Modulated Interferometer

- Utilizes conventional, low-cost heterodyning architecture
- Correlation occurs in the digital domain
- Governed by simple fundamental parameter relations:

  Antenna baseline, $D$ \quad $f_\omega, f_\theta \propto D$

  Carrier wavelength, $\lambda$ \quad $f_\omega, f_D \propto 1/\lambda$

  Chirp-rate, $K$ \quad $f_\theta, f_R \propto K$
Simulation Configuration
Validation of Theory - Linear Frequency Modulated Interferometer

- Initial validation performed using simulation
- Simulation parameters:
  - \( f_0 \in \{5.8, 11.6\} \text{ GHz}, \beta = 100 \text{ MHz}, \tau \in \{100, 200\} \mu\text{s} \)
  - \( D \in \{10, 20\} \cdot \lambda, R = 10 \text{ m} \)
  - \(|\omega| = \pi/2 \text{ rad} \cdot \text{s}^{-1}, |\gamma| = 5\pi/2 \text{ rad} \cdot \text{s}^{-2} \)
  - Parameters chosen to be replicate able with physical hardware

Simulated target trajectory
Simulation Results
Validation of Theory - Linear Frequency Modulated Interferometer

\[ f_0 = 5.8 \text{ GHz}, \beta = 100 \text{ MHz}, \tau = 200 \mu\text{s}, D = 10 \cdot \lambda \]

Angle RMSE: 0.0377 rad  Ang. Vel. RMSE: 0.1784 rad \cdot s^{-1}
**Simulation Results**

*Validation of Theory - Linear Frequency Modulated Interferometer*

\[ f_0 = 5.8 \text{ GHz}, \beta = 100 \text{ MHz}, \tau = 200 \mu\text{s}, D = 20 \cdot \lambda \]

*Angle RMSE: 0.0224 rad  Ang. Vel. RMSE: 0.1613 rad \cdot s^{-1}*

![Diagram](image1)

![Diagram](image2)
Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

\[ f_0 = 5.8 \text{ GHz}, \beta = 100 \text{ MHz}, \tau = 100 \mu s, D = 10 \cdot \lambda \]

Angle RMSE: 0.0767 rad    Ang. Vel. RMSE: 0.1760 rad \( \cdot \) s\(^{-1} \)
Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

\[ f_0 = 11.6 \text{ GHz}, \beta = 100 \text{ MHz}, \tau = 200 \mu s, D^* = 20 \cdot \lambda \]

\[ \text{Angle RMSE: 0.0316 rad} \quad \text{Ang. Vel. RMSE: 0.1550 rad} \cdot \text{s}^{-1} \]
Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

\[
\begin{align*}
\text{Angle RMSE: } & 0.0377 \text{ rad} \\
\text{Ang. Vel. RMSE: } & 0.1784 \text{ rad} \cdot \text{s}^{-1}
\end{align*}
\]

\[
\begin{align*}
\text{Angle RMSE: } & 0.0224 \text{ rad} \\
\text{Ang. Vel. RMSE: } & 0.1613 \text{ rad} \cdot \text{s}^{-1}
\end{align*}
\]

\[
\begin{align*}
\text{Angle RMSE: } & 0.0767 \text{ rad} \\
\text{Ang. Vel. RMSE: } & 0.1760 \text{ rad} \cdot \text{s}^{-1}
\end{align*}
\]

\[
\begin{align*}
\text{Angle RMSE: } & 0.0316 \text{ rad} \\
\text{Ang. Vel. RMSE: } & 0.1550 \text{ rad} \cdot \text{s}^{-1}
\end{align*}
\]
Conclusions

• New radar architecture and equation derivations for:
  • Direct, simultaneous measurement of **angle and angular velocity** using an **LFM waveform** and **correlation interferometry** for a point-target
  • Less complex than a dense, beamforming arrays typically used for angle estimation

• Simulated validation of:
  • Simultaneous **direct measurement of angle of arrival and angular velocity** of a point-target using LFM waveform using a simple process analogous to range-Doppler processing
Questions?

Email: merlojas@msu.edu
Backup Slides
Six Degree of Freedom Measurements

The Interferometric Approach

Position

\[ R = ||P|| \approx -f_{bn} \frac{c}{2K} \]

\[ \theta_i \approx \sin^{-1} \left( -\frac{\tau}{\beta} \frac{c}{D_s} \right); \ i \in \{x, y\} \]

\[ \theta = \text{atan2} (\tan \theta_x, \sin \phi) \quad \text{or} \quad \theta = \text{atan2} (\tan \theta_y, \cos \phi) \]

\[ \phi = \text{atan2} (F(\theta_y), F(\theta_x)); \ \theta_i \in [-\pi/2, \pi/2] \]

where \[ F(\alpha) = \frac{R \tan \alpha}{\sqrt{\tan^2 \theta_x \tan^2 \theta_y + \tan^2 \theta_x + \tan^2 \theta_y}} \]
Six Degree of Freedom Measurements

The Interferometric Approach

Velocity

\[ v_R = -\frac{f_d \lambda}{2\tau} \quad v_\theta_i = f_\omega \frac{R}{D_\lambda \tau}; \quad i \in \{x, y\} \]

Movement:

\[ \| \mathbf{V} \| = \sqrt{v_R^2 + v_{\theta_x}^2 + v_{\theta_y}^2} \quad \theta_v = \text{atan2} \left( v_\phi, v_R \right) \]

\[ v_\phi = \sqrt{v_{\theta_x}^2 + v_{\theta_y}^2} \quad \phi_v = \text{atan2} \left( v_{\theta_x}, v_{\theta_y} \right) \]
Current Methods
State of the Art

Current Methods

Current radars **only perform direct estimates** of:

- **Range**
  - Phase interferometry, waveform modulation (AM, FM, PM)
- **Range-rate**
  - Doppler shift
- **Angle**
  - Mechanical scanning, amplitude comparison, FDoA, TDoA, beamforming, correlative interferometry

Modern radars apply a **locate and track** method **to derive angular-rate**
Radial Velocity Measurement

Current Methods

- Doppler is used for direct velocity measurement based on frequency shift
- Can be employed for continuous-wave or modulated waveforms with periodic, stationary phase points

\[
\begin{align*}
  s_t(t) &= \exp\left(j2\pi f_0 t\right) \\
  r(t) &= \exp\left[j2\pi f_0 (t - \tau_d)\right]
\end{align*}
\]

where \( \tau_d = \frac{2R}{c} \)

\[
\begin{align*}
  r_d(t) &= r(t) \cdot s_t^*(t) \\
  &= \exp\left(-j2\pi f_0 \tau_d\right)
\end{align*}
\]
Radial Velocity Measurement

Current Methods

\[ r_d(t) = \exp \left( -j2\pi f_0 \tau_d \right) \]

Doppler-shift found by differentiation of phase of \( r_d(t) \)

\[ f_d(t) = \frac{1}{2\pi} \frac{d\phi_{r_d}(t)}{dt} = -\frac{d}{dt} f_0 \tau_d \]

Because \( R \) is time-dependent,

\[ \frac{d}{dt} \tau_d = \frac{2v_R}{c} \]

\[ f_d(t) = -\frac{2v_R}{\lambda} \quad \Rightarrow \quad v_R = -\frac{f_d \lambda}{2} \]

where \( \lambda \) is the wavelength

Bistatic Radar
Range-Doppler Measurement

Current Methods

• With modulation, range and velocity can be obtained

• Linear frequency modulation (LFM) is commonly used due to its ease of implementation

\[ \omega_I(t) = 2\pi (f_0 + Kt); \quad t \in [-\tau/2, \tau/2] \]

where  \( K = \beta / \tau \) is the chirp rate  
\( \beta \) is the chirp bandwidth  
\( \tau \) is the chirp duration

\[ s_I(t) = A(\theta) \exp \left[ \int \omega_s(t)dt \right] = A(\theta) \exp \left[ j2\pi \left( f_0 t + \frac{K}{2} t^2 \right) \right] \]
Range-Doppler Measurement

**Current Methods**

\[ s_t(t) = A(\theta) \exp \left[ j2\pi \left( f_0 t + \frac{K}{2} t^2 \right) \right] \]

Received signal at \( r_n \):

\[ r_n = A(\theta) \exp \left\{ j2\pi \left[ f_0 (t - \tau_{dn}) + \frac{K}{2} (t - \tau_{dn})^2 \right] \right\} \]

Downconverted signal at \( r_{bn} \):

\[ r_{bn}(t) = r_n(t) \cdot s_i^*(t) \]

\[ = A(\theta) \exp \left\{ -j2\pi \left[ f_0 \tau_{dn} + \frac{K}{2} \left( \tau_{dn}^2 - 2\tau_{dn}t \right) \right] \right\} \]

\[ \text{where } \tau_{dn} = \frac{2R_n}{c} \]
Range-Doppler Measurement

**Current Methods**

\[ r_{bn}(t) = A(\theta) \exp \left\{ -j2\pi \left[ f_0 \tau_{dn} + \frac{K}{2} \left( \tau_{dn}^2 - 2\tau_{dn}t \right) \right] \right\} \]

Beat frequency found by differentiation of phase of \( r_{bn} \)

\[ f_{bn} = \frac{1}{2\pi} \frac{d\phi_{r_{bn}}(t)}{dt} = -\frac{2v_R}{\lambda} - \frac{2}{c} K \left( R + v_R t - \frac{4}{c^2} R v_R \right) \]

\( \ll \) range term

\[ \rightarrow R = -f_{bn} \frac{c}{2K} \quad \text{for} \quad v_r = 0 \]
Range-Doppler Measurement

Current Methods

\[ r_{bn}(t) = A(\theta) \exp \left\{ -j2\pi \left[ f_0 \tau_{dn} + \frac{K}{2} \left( \tau_{dn}^2 - 2\tau_{dn}t \right) \right] \right\} \]

Beat frequency found by differentiation of phase of \( r_{bn} \)

\[ f_{bn} = \frac{1}{2\pi} \frac{d\phi_{r_{bn}}(t)}{dt} = -\frac{2v_R}{\lambda} - \frac{2}{c} K \left( R + v_R t - \frac{4}{c^2} R v_R \right) \]

\[ \Rightarrow R \approx -f_{bn} \frac{c}{2K} \]

under quasi-static assumption
Range-Doppler Measurement

Current Methods

Doppler shift from moving LFM scatterer:

\[ f_d = \frac{1}{2\pi} \Delta b_n(t) = f_0 \Delta \tau_{dn} + \frac{K}{2} \left[ \tau_{dn_1}^2 - \tau_{dn_2}^2 - 2\Delta \tau_{dn}t \right] \]

where \( \Delta \tau_{dn} = \frac{2}{c} (R_2 - R_1) = -\frac{2}{c} v_{Rn} \tau \) and \( v_{Rn} = -\frac{R_2 - R_1}{\tau} \)

\[ = -\frac{2v_R}{\lambda} \tau \implies v_R = -\frac{f_d\lambda}{2\tau} \]

- If the PRF ≥ Nyquist frequency of the Doppler shift, the velocity can be resolved in the slow-time
Measurement System — Radar Hardware

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

- Transmitter:
  - 40.5 GHz continuous-wave
- Antennas:
  - TX: 15 dBi
  - RX: 10 dBi
  - $L = 7\lambda$
- ADC:
  - National Instruments USB-6002 DAQ
  - Sample Rate ($f_s$): 4.166 kHz
- Two experimental configurations:
  - Varying bearing angle
  - Varying elevation angle
Varying Bearing - Velocity Estimates

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Estimated Velocity Vs. Target Bearing

Estimated Velocity Error Vs. Target Bearing
Varying Bearing - Bearing Estimates

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Estimated Target Bearing

Estimated Target Bearing Error

Estimated Velocity - Estimation vs. True Velocity

Estimated Velocity Error - Estimation vs. True Velocity
Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer
Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Varying $\beta$: $\beta = 0^\circ; \phi = 0^\circ$

Correlator Response ($\Phi = 0$)

Correlator Response ($\Phi = 90$)

Correlator Response ($\Phi = 45$)

Doppler Response (RX1)
Elevation Estimate Configuration

*Experimental Validation - Dual-Axis Continuous-Wave Interferometer*

**Varying β:** \( \beta = 10^\circ; \ \phi = 0^\circ \)

![Correlator Response (\( \Phi = 0 \))]  
![Correlator Response (\( \Phi = 90 \))]  
![Correlator Response (\( \Phi = 45 \))]  
![Doppler Response (RX1)]
Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Varying $\beta$: $\beta = 20^\circ$; $\phi = 0^\circ$

Correlator Response ($\Phi = 0$)

Correlator Response ($\Phi = 90$)

Correlator Response ($\Phi = 45$)

Doppler Response (RX1)
Varying $\beta$: $\beta = 30^\circ; \phi = 0^\circ$

Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Correlator Response ($\Phi = 0$)

Correlator Response ($\Phi = 90$)

Doppler Response (RX1)

Correlator Response ($\Phi = 45$)
Elevation Estimate Configuration

*Experimental Validation - Dual-Axis Continuous-Wave Interferometer*

**Varying β:** \( β = 40°; \ φ = 0° \)

Correlator Response (\( Φ = 0 \))

Correlator Response (\( Φ = 90 \))

Correlator Response (\( Φ = 45 \))

Doppler Response (RX1)
Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Varying $\beta$: $\beta = 45^\circ; \phi = 0^\circ$

Correlator Response ($\Phi = 0$)

Correlator Response ($\Phi = 90$)

Correlator Response ($\Phi = 45$)

Doppler Response (RX1)
Varying Elevation - Velocity Estimates
Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Doppler Velocity Vs. Offset Angle $\beta$

Offset Angle $\beta$

Velocity (mm $\cdot$ s$^{-1}$)

- Measured
- $V \cdot \sin(\beta)$

Doppler Estimate Error Vs. Offset Angle $\beta$

Offset Angle $\beta$

Velocity (mm $\cdot$ s$^{-1}$)

-22.12
-43.12
2.19
-5.65

Varying Elevation - Velocity Estimates

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Tangent Velocity Vs. Offset Angle Vs. Baseline Angle

Tangent Velocity Estimate Error Vs. Offset Angle $\beta$
Varying Elevation - Velocity Estimates

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

3D Velocity Estimate Vs. Offset Angle $\beta$

3D Estimate Error Vs. Offset Angle $\beta$

<table>
<thead>
<tr>
<th>Offset Angle</th>
<th>Measured</th>
<th>Measured (fall-off)</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12.86</td>
<td>12.18</td>
<td>-160.00</td>
</tr>
<tr>
<td>5</td>
<td>-44.54</td>
<td>-45.42</td>
<td>-109.45</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>