High Accuracy Wireless Time Synchronization for Distributed Antenna Arrays

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Outline

1. Distributed Antenna Arrays Overview
2. High Accuracy Time Transfer
3. Experimental Results
Distributed Array Applications and Benefits

**Reduced Cost**
- Smaller, low-cost platforms
- Cost distributed over many nodes

**Reconfigurable**
- Adaptable sub-arrays to meet *bandwidth* and *spatial* requirements

**Increased Robustness**
- Nodes may be added or removed without failure of array

**Increased Gain**
- Transmission gain \( \propto N^2 \)
- Reception gain \( \propto M \)
- Total gain \( \propto N^2 M \)
Distributed Array Coordination Challenges

Frequency Syntonization

\[ s_1 : f_1 = 5.0 \text{ Hz} \]

\[ s_2 : f_2 = 6.0 \text{ Hz} \]

Phase Alignment

\[ s_1 : \phi_1 = 0 \text{ rad} \]

\[ s_2 : \phi_2 = \frac{\pi}{2} \text{ rad} \]

Time Synchronization

\[ s_1 : \psi_1 = 0 \text{ rad} \]

\[ s_2 : \psi_2 = \pi \text{ rad} \]

Focus of this work
Distributed Antenna Arrays Overview

Distributed Array Time Error Tolerance

Probability of coherent gain:

\[ P(G_c \geq X) \]

where

\[ G_c = \frac{|s_r s^*_i|}{|s_1 s^*_i|} \]

- \( s_r \): received signal
- \( s_1 \): ideal signal

\[ P(G_c \geq X) \geq 0.9 \]

Timing error <10% pulse duration

Modulation requires stricter timing

\[ \sigma_T/T \]


Distributed Antenna Arrays Overview

Distributed Array Coordination Topologies

**Closed-Loop**

**Pros**
- Minimal information sharing required
- Channel errors corrected implicitly

**Cons**
- Can only transmit to base station (no beamsteering)
- Time consuming due to potentially large number of iterations

**Open-Loop**

**Pros**
- Compatible with noncooperative/passive targets
- Arbitrary beamforming capability

**Cons**
- Stringent inter-node coordination requirements
- Channel errors to target location not inherently corrected
System Time Model

- Time at node $n$:
  \[ T_n(t) = t + \delta_n(t) + \nu_n(t) \]

  - $t$: true time
  - $\delta_n(t)$: time-varying bias
    - Assumed quasi-static over synchronization epoch
    - No further assumptions on distribution of $\delta_n$
  - $\nu_n(t)$: other zero-mean noise sources
  - $\Delta_{0n}(t) = \delta_0(t) - \delta_n(t)$

- Goal: estimate and compensate for $\delta_n$
**2 | High Accuracy Time Transfer**

## Time Transfer Techniques

### One-Way Time Transfer

**Pros**
- Receiver nodes do not need to transmit
- One node can provide time to many anonymous receivers

**Cons**
- Channel must be well characterized to accurately determine and subtract propagation delay
- Positions and trajectories of nodes must be known

### Two-Way Time Transfer

**Pros**
- Both nodes implicitly estimate channel delay
- Both nodes determine their relative offset

**Cons**
- Requires all nodes to have transmitters
- Time transfer must be performed pairwise
  - \( N \) orthogonal channels required to synchronize \( N \) nodes

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\( \tau \), \( v \)
Two-Way Time Transfer Synchronization

- Assumptions
  - Channel is quasi-static over synchronization epoch

- Propagation delay estimate:
  \[ \tau_{0n} = \frac{(t_{RX0} - t_{TXn}) + (t_{TX0} - t_{RXn})}{2} \]

- Timing skew estimate:
  \[ \Delta_{0n} = \frac{(t_{RX0} - t_{TXn}) - (t_{TX0} - t_{RXn})}{2} \]

High Accuracy Time Delay Waveform

- The delay accuracy lower bound (CRLB) for time is given by
  \[ \text{var}(\hat{\tau} - \tau) \geq \frac{1}{2\xi_f^2} \cdot \frac{N_0}{E_s} \]
  - \( \xi_f^2 \): mean-squared bandwidth
  - \( N_0 \): noise power spectral density
  - \( E_s \): signal energy
    \[ \frac{E_s}{N_0} = \tau_p \cdot \text{SNR} \cdot \text{NBW} \]
  - \( \tau_p \): integration time
  - \( \text{SNR} \): signal-to-noise ratio
  - \( \text{NBW} \): noise bandwidth

2 | High Accuracy Time Transfer

High Accuracy Time Delay Waveform

\[
\text{var}(\hat{\tau} - \tau) \geq \frac{1}{2\zeta_f^2} \cdot \frac{N_0}{E_s}
\]

- For constant-SNR, maximizing \( \zeta_f^2 \) will yield improved delay estimation

\[
\zeta_f^2 = \int_{-\infty}^{\infty} (2\pi f)^2 |G(f)|^2 df
\]

- \( \zeta_f^{(LFM)} = (\pi \cdot \text{BW})^2 / 3 \)
- \( \zeta_f^{(two-tone)} = (\pi \cdot \text{BW})^2 \)

\[\text{Relative Mean Sq. Bandwidth} \begin{pmatrix} \zeta_f^2 \zeta_f^0 \end{pmatrix}\]

\[\text{Fractional Occupied Bandwidth (\( \phi \))}\]

2 | High Accuracy Time Transfer

Delay Estimation and Refinement

- Discrete matched filter (MF) used in initial time delay estimate

\[ s_{MF}[n] = s_{RX}[n] \otimes s_{TX}^*[-n] = \mathcal{F}^{-1}\{S_{RX}S_{TX}^*\} \]

- Two-tone matched filter waveform is highly ambiguous

- High SNR or narrow-band pulse required to disambiguate peaks

Delay Estimation and Refinement

- MF causes estimator bias due to time discretization

- Refinement of MF obtained using Quadratic Least Squares (QLS) fitting to find true delay based on three sample points

\[ \hat{t} = \frac{T_s}{2} \frac{s_{\text{MF}}[n_{\text{max}} - 1] - s_{\text{MF}}[n_{\text{max}} + 1]}{s_{\text{MF}}[n_{\text{max}} - 1] - 2s_{\text{MF}}[n_{\text{max}}] + s_{\text{MF}}[n_{\text{max}} + 1]} \]

where

\[ n_{\text{max}} = \arg\max_n \{s_{\text{MF}}[n]\} \]

Delay Estimation and Refinement

- QLS results in small residual bias due to an imperfect representation of the underlying MF output.
- Residual bias is a function of waveform and sample rate.
- Can be easily corrected via lookup table.
Experimental Results

Experimental Time Synchronization Setup

- **Time Transfer Waveform**
  - \( f_c = 5.9 \text{ GHz} \)
  - \( \text{BW} = 50 \text{ MHz} \) (tone separation)
  - \( \tau_{\text{rise-fall}} = 50 \text{ ns} \) (rise-fall time)
  - \( \tau_p = 10 \mu\text{s} \) (pulse duration)
  - \( \tau_{\text{sync}} = 50.01 \text{ ms} \) (synchronization epoch)

- **Antenna**
  - 5.9 GHz, 13.2 dBi Yagi-Uda antennas

- **SDR**
  - \( f_s = 200 \text{ MSa/s} \) (sample rate)
Experimental Results

Experimental Time Synchronization Setup
Time-Transfer and Beamforming Precision

- Time transfer st. dev. was estimated on SDR using time update deltas.
- Beamforming st. dev. was estimated by cross-correlating received waveforms on oscilloscope.
- Inter-channel bias was <10 ps after calibration.
Conclusions

• Using spectrally-sparse two-tone pulses, theoretical maximum time-delay estimation may be achieved

• Approach experimentally validated using two-way time synchronization on software-defined radios

• Using a 50 MHz signal bandwidth, precisions of
  • ~2 ps for two-way time transfer
  • ~7 ps for beamforming

are achieved

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Questions?

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