High Accuracy Wireless Time-Frequency Transfer for Distributed Phased Array Beamforming

Jason M. Merlo*, Anton Schlegel, and Jeffrey A. Nanzer
Electrical and Computer Engineering
Michigan State University, USA
Distributed Phased Arrays

Next Generation Satellite Cellular Networks

Distributed V2X Sensing

Space Communication and Remote Sensing

Earth-Based communication base station

Sensing and communication relay satellite constellation

Receiver or imaging target

Precision Agricultural Sensing

Single-platform resolution

Distributed array resolution
Distributed Array Synchronization

Node: 1

\( N_1 \)

Node: 2

\( N_2 \)

Target

Frequency Syntonization

\[ s_1 : f_1 = 5.0 \text{ Hz} \]

\[ s_2 : f_2 = 6.0 \text{ Hz} \]

Phase Alignment

\[ s_1 : \phi_1 = 0 \text{ rad} \]

\[ s_2 : \phi_2 = \frac{3\pi}{2} \text{ rad} \]

Time Synchronization

\[ s_1 : \psi_1 = 0 \text{ rad} \]

\[ s_2 : \psi_2 = \frac{\pi}{3} \text{ rad} \]

\[ s_1 + s_2 \]

\[ s_1 + s_2 = \sum_{n=1}^{2} \alpha_n (t - \delta t_n) \exp\{j[2\pi(f + \delta f_n) + \phi_n]\} \]
Distributed Array Performance

Probability of coherent gain:

\[ P(G_c \geq X) \]

where

\[ G_c = \frac{|s_r s_r^*|}{|s_i s_i^*|} \]

- \( s_r \): received signal
- \( s_i \): ideal signal

Barker Code (13-bit)

Linear Frequency Modulated

Timing error <10% pulse duration


System Time Model

- Local time at node $n$:

$$T_n(t) = t + \delta_n(t) + \nu_n(t)$$

- $t$: true time
- $\delta_n(t)$: time-varying offset from global true time
- $\nu_n(t)$: other zero-mean noise sources
- $\Delta_{0n}(t) = T_0(t) - T_n(t)$

- Goal: estimate and compensate for $\Delta_{0n}$
Achieving Synchronization

Two-Way Time Synchronization

• **Assumptions:**
  – Link is quasi-static and reciprocal during the synchronization epoch

• Timing skew estimate:

\[
\Delta_{0n} = \frac{(T_{RX0} - T_{TXn}) - (T_{RXn} - T_{TX0})}{2}
\]

For compactness of notation: \(T_m(t_{TXn}) = T_{TXn}\)
The delay accuracy lower bound (CRLB) for time is given by
\[ \text{var}(\hat{\tau} - \tau) \geq \frac{1}{2\zeta_f^2} \cdot \frac{N_0}{E_s} \]

- \( \zeta_f^2 \): mean-squared bandwidth
- \( N_0 \): noise power spectral density
- \( E_s \): signal energy

\[ \frac{E_s}{N_0} = \tau_p \cdot \text{SNR} \cdot \text{NBW} \]

- \( \tau_p \): integration time
- \( \text{SNR} \): signal-to-noise ratio
- \( \text{NBW} \): noise bandwidth

High Accuracy Delay Estimation

\[
\text{var}(\hat{t} - \tau) \geq \frac{1}{2\zeta_f^2} \cdot \frac{N_0}{E_s}
\]

- For constant-SNR, maximizing \( \zeta_f^2 \) will yield improved delay estimation

\[
\zeta_f^2 = \int_{-\infty}^{\infty} (2\pi f)^2 |G(f)|^2 df
\]

- \( \zeta_f^{2(LFM)} = (\pi \cdot \text{BW})^2 / 3 \)
- \( \zeta_f^{2(\text{two-tone})} = (\pi \cdot \text{BW})^2 \)

Delay Estimation Refinement

- Discrete matched filter (MF) used in initial time delay estimate

\[ s_{MF}[n] = s_{RX}[n] \ast s_{TX}^*[-n] \]

\[ = \mathcal{F}^{-1}\{S_{RX}S_{TX}^*\} \]

- Two-tone matched filter waveform is highly ambiguous

- High SNR or narrow-band pulse required to disambiguate peaks
Delay Estimation Refinement

- MF causes estimator bias due to time discretization
- Refinement of MF obtained using Quadratic Least Squares (QLS) fitting to find true delay based on three sample points

\[
\hat{\tau} = T_s \cdot \frac{s_{MF}[n_{\text{max}} - 1] - s_{MF}[n_{\text{max}} + 1]}{2 \cdot s_{MF}[n_{\text{max}} - 1] - 2s_{MF}[n_{\text{max}}] + s_{MF}[n_{\text{max}} + 1]}
\]

where

\[
n_{\text{max}} = \arg\max_{n} \{s_{MF}[n]\}\]
System Configuration

Legend:
- Time Transfer Waveform
- Frequency Transfer Waveform
- 10 MHz Frequency Reference
- PPS (coarse time sync)
- 10 GbE (data)
- Beamforming

Node 0:
- GNSS
- Desktop Computer
- Signal Generator
- Two-Tone Frequency Locking Circuit

Node 1:
- GNSS

SDR 0:
- TX0
- RX0
- REF_IN
- REF_OUT
- 10GbE

SDR 1:
- TX1
- RX1
- PPS_IN
- REF_IN
- 10GbE

Oscilloscope:
- CH-1
- Target

10 MHz Frequency Reference:
- PPS (coarse timesync)

Manualy muted ~100s of milliseconds

1.0 GHz
4.3 GHz
5.5 GHz
2–5 m
41 m

IEEE MTT-S

Tu2C-3
System State Flow

Start → Initial PPS Sync

- Coarse alignment ~10 ns

Initial PPS Sync → TWTT Exchange → Update $\Delta_{0n}$

- Residual bias compensated to picosecond level

TWTT Exchange → Transmit beamforming pulses

- Compensate using $\Delta_{0n}$, and beamsteer using $\tau_{bf}$ and $\phi_{bf}$

Transmit beamforming pulses → Estimate $\tau_{bf}$ and $\phi_{bf}$ at target

First Pulse?

- No → TWTT Exchange
- Yes → Save $\tau_{bf}$ and $\phi_{bf}$ as calibration

Save $\tau_{bf}$ and $\phi_{bf}$ as calibration → System State Flow
Beamforming Waveforms

- Each node transmitted orthogonal LFMs followed by two CW pulses

**Diagram:**
- **Node 0**
- **Node 1**

**Annotations:**
- LFM: Time/Phase Estimation
- CW Pulses: Coherent Gain Estimation
Beamforming Experiment

Transmit Nodes Setup

Target Node Setup (41 m downrange)
Beamforming Experiment

- Time Transfer SNR: ~23dB
- Cycle Time: ~4 s
- Tested Frequency Transfer Failure

- Beamforming Std.: $18 < \sigma_{bf} < 40$ ps
- Phase Std.: $0.04\pi < \sigma_{\phi} < 0.10\pi$
- Throughput (BPSK): $2.5 < b_{max} < 5.5$ Gbps
- Max Carrier Freq.: $1.0 < f_{max} < 2.78$ GHz
Beamforming Experiment

Coherent Gain: 0.91

Nodes 0+1

Coherent Gain: 0.95

Nodes 0+1

Coherent Gain: 0.94

Nodes 0+1
Conclusion

• Discussed:
  – High-accuracy time transfer technique using spectrally sparse two-tone waveforms
  – Two-step refinement and QLS bias compensation process

• Demonstrated:
  – fully wireless outdoor time-frequency synchronization and beamforming with $G_c > 0.9$ over a 41 m

<table>
<thead>
<tr>
<th>Internode Distance</th>
<th>Min. Time Transfer Std.</th>
<th>Min. Beamforming Std.</th>
<th>Max. Throughput (BPSK; $G_c \geq 0.9$)</th>
<th>Max. Carrier Frequency $P(G_c \geq 0.9) \geq 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 m</td>
<td>10.47 ps</td>
<td>18.00 ps</td>
<td>5.56 Gbps</td>
<td>2.78 GHz</td>
</tr>
<tr>
<td>5.0 m</td>
<td>14.79 ps</td>
<td>24.02 ps</td>
<td>4.16 Gbps</td>
<td>2.08 GHz</td>
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</tbody>
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Questions

Thank you to our project sponsors and collaborators:
Backup Slides
Delay Estimation Refinement

- QLS results in small residual bias due to an imperfect representation of the underlying MF output.
- Residual bias is a function of waveform and sample rate.
- Can be easily corrected via lookup table.
Baseline 5.0 m

Transmit Nodes Setup

Target Node Setup (41 m downrange)
Baseline 5.0 m

- Time Transfer SNR: ~23dB
- Cycle Time: ~4 s
- Beamforming Std.: $24 < \sigma_{bf} < 100$ ps
- Phase Std.: $0.05\pi < \sigma_{\phi} < 0.10\pi$
- Throughput (BPSK): $1.0 < b_{\text{max}} < 4.16$ Gbps
- Max Carrier Freq.: $0.67 < f_{\text{max}} < 2.08$ GHz
Baseline 5.0 m

Coherent Gain: 0.90

Coherent Gain: 0.92

Coherent Gain: 0.94

Nodes 0+1

Node 0

Node 1

Nodes 0+1

Node 0

Node 1

Nodes 0+1

Node 0

Node 1