Joint Measurement of Target Angle and Angular Velocity Using Interferometric Radar with FM Waveforms

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Distributed Electromagnetics Theory and Applications

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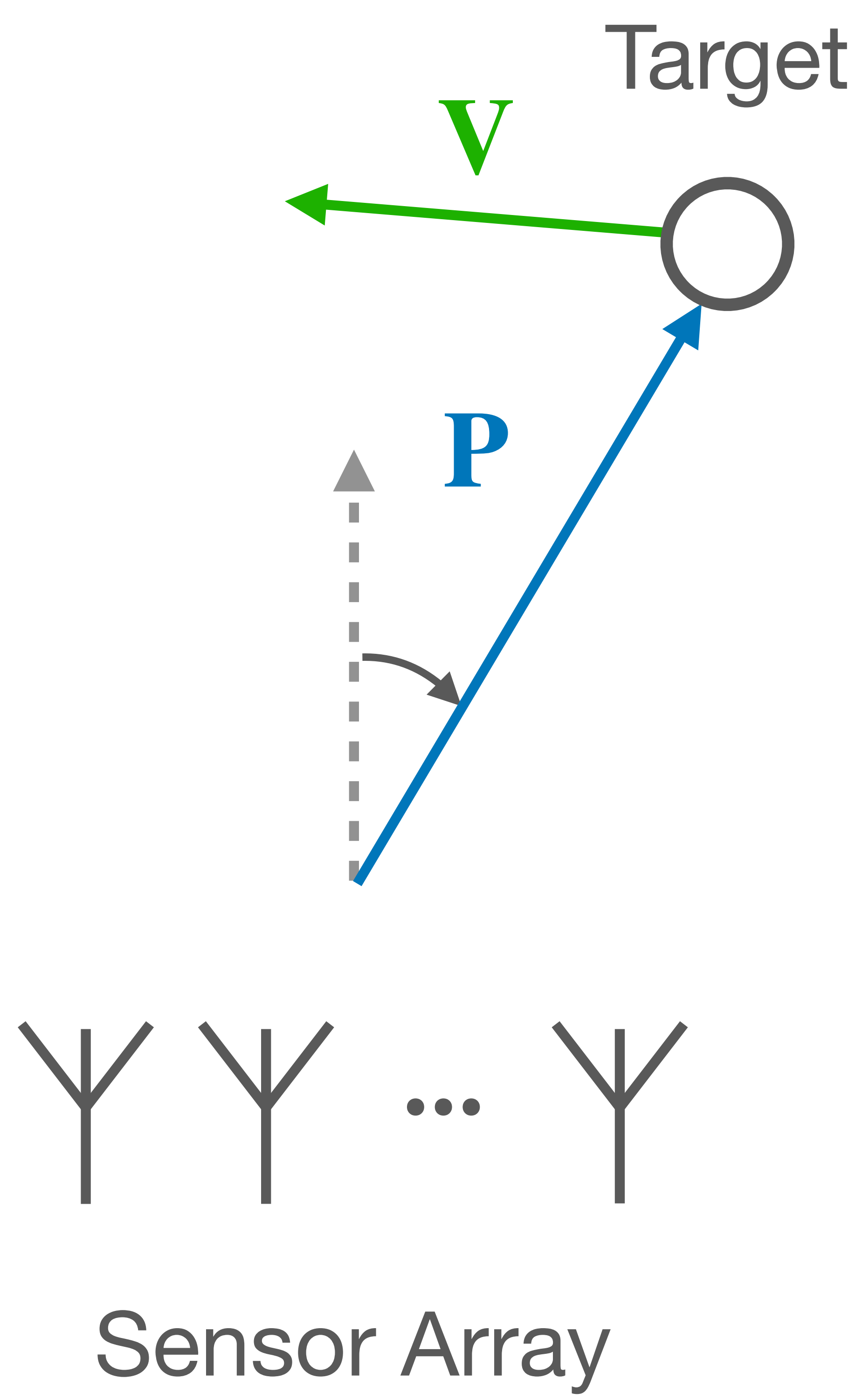
Motivation

Angle and Angular Rate Measurement

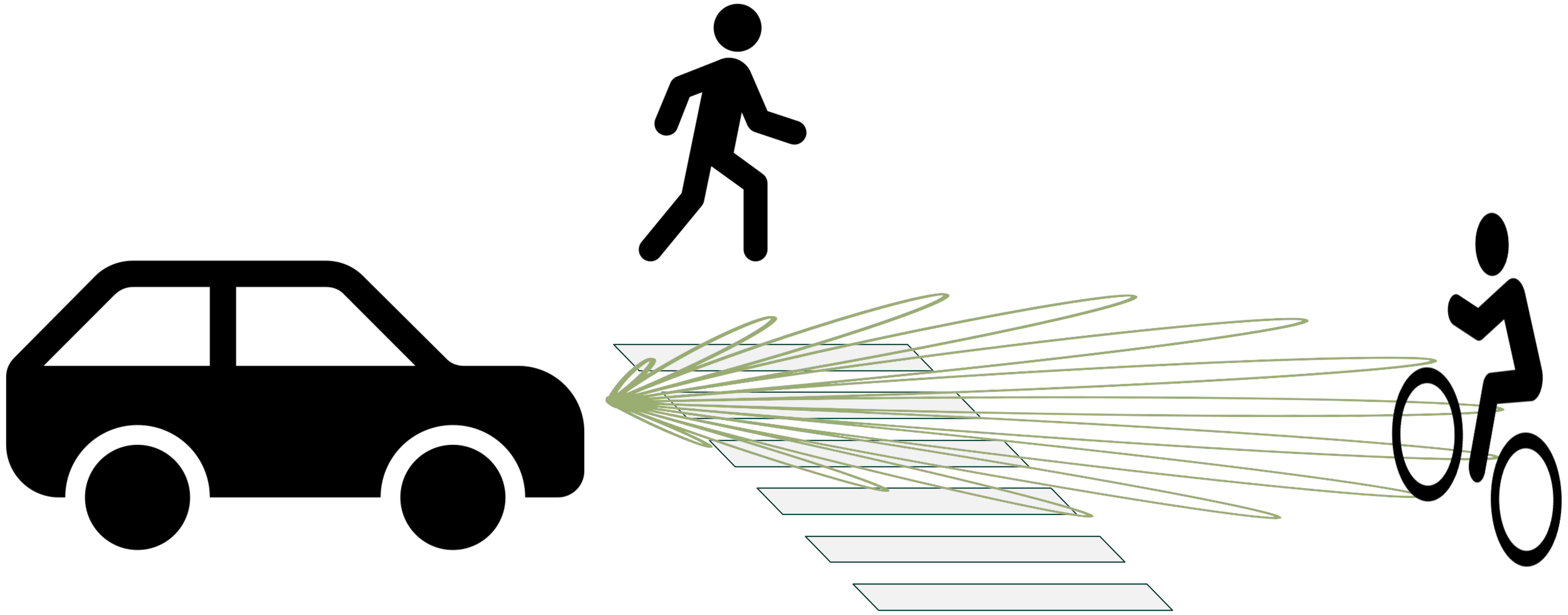
- **Angular rate is not directly measured** by conventional radar systems; instead a **locate and track method is typically used** to derive it
- Moreover common array angle estimations methods are complex and computationally expensive
- **Active correlation interferometry can be used to measure angle and angular rate** using direct frequency estimation, **analogously to range-Doppler measurement**

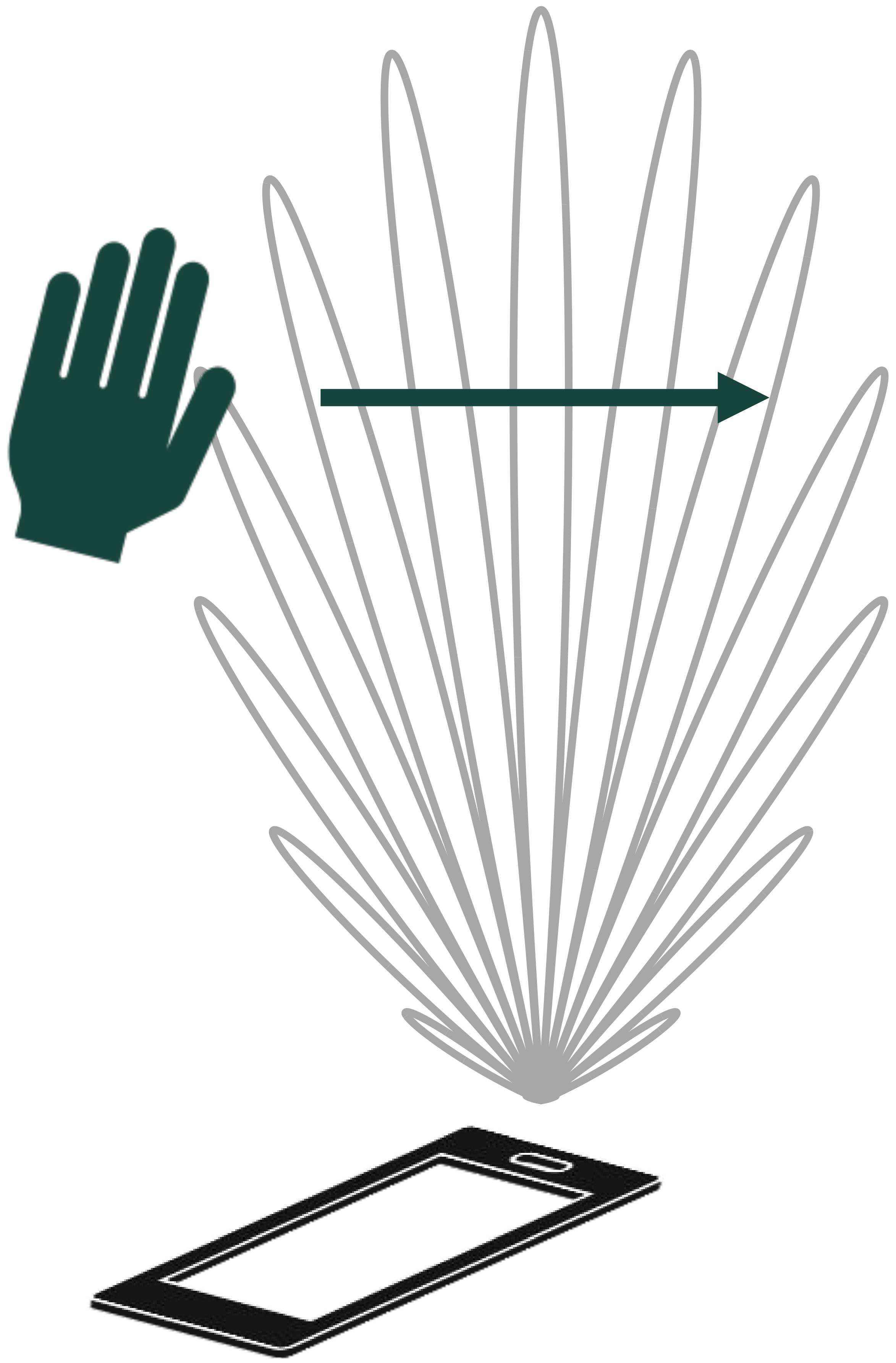


Applications

Angle and Angular Rate Measurement

- Human Computer Interfaces (HCI)
- Automotive Radar
- Airspace Monitoring
- Space-object Monitoring

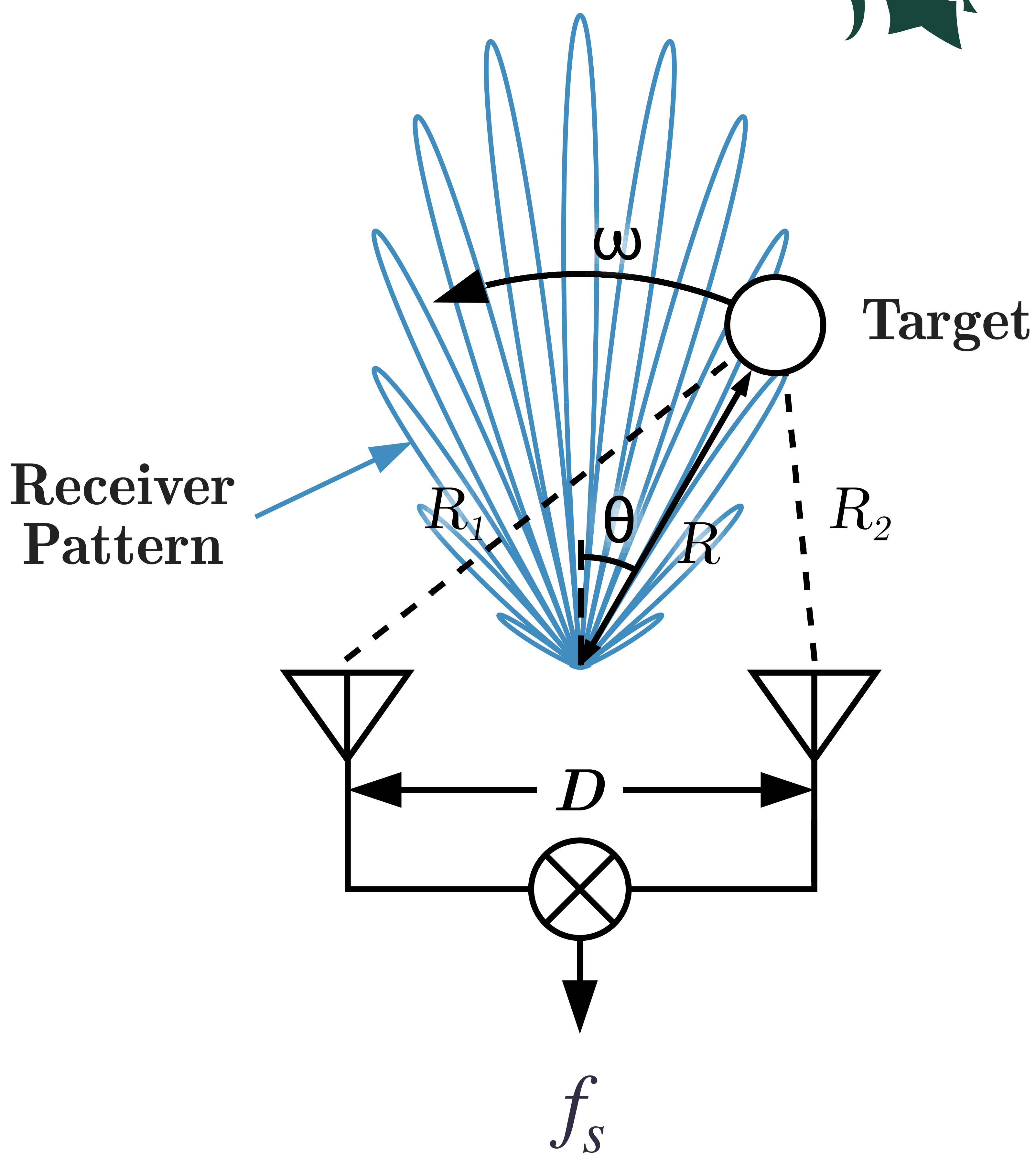




Background

Correlation Interferometry

- Correlation interferometry can be used to measure angular rate of a target[1]
- Phases across multiple apertures are summed; this produces an interference or “fringe” pattern
- Moving targets produce frequency proportional to angular rate after correlation
- Using *active* correlation interferometry both angular rate and angle can be estimated



Correlation Interferometer

(1) J. A. Nanzer, “Millimeter-wave interferometric angular velocity detection,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 12, pp. 4128–4136, Dec 2010.

The Interferometric Approach

Angular Rate Measurement

The Interferometric Approach

Correlator output:

$$r_c(t) = A(\theta)\exp\left(j2\pi f_0 \tau_g\right) = A(\theta)\exp\left(j2\pi D_\lambda \sin\theta\right)$$

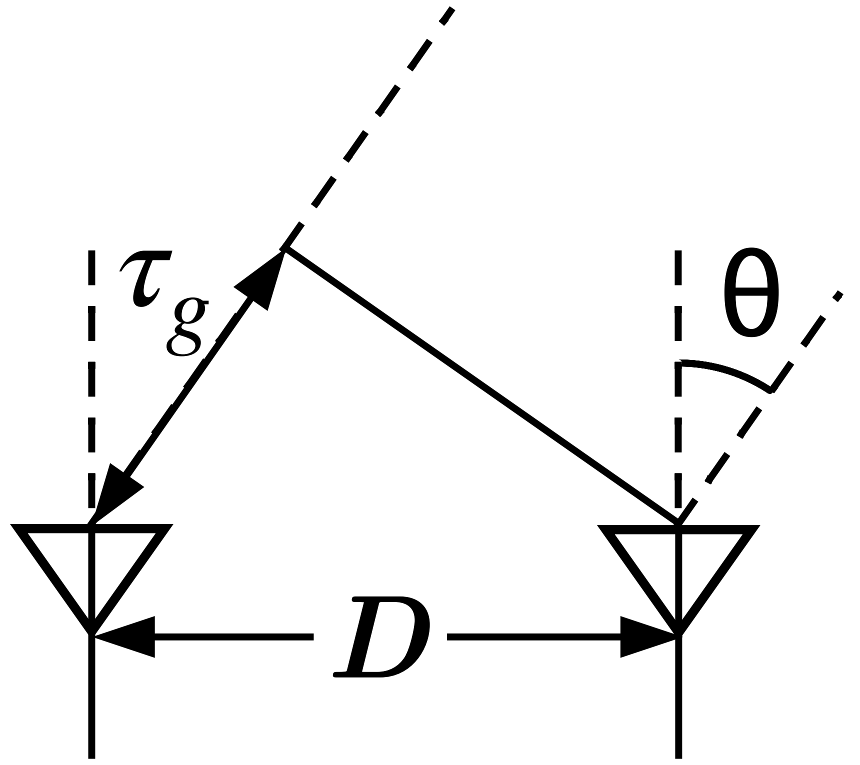
Angular rate measurement:

Using $\omega = \dfrac{d\theta}{dt} \quad \Rightarrow \quad \theta = \omega t + \theta_0$

$$f_\omega = \frac{1}{2\pi}\frac{d\phi_{r_c}(t)}{dt} = \omega D_\lambda \cos(\theta)$$

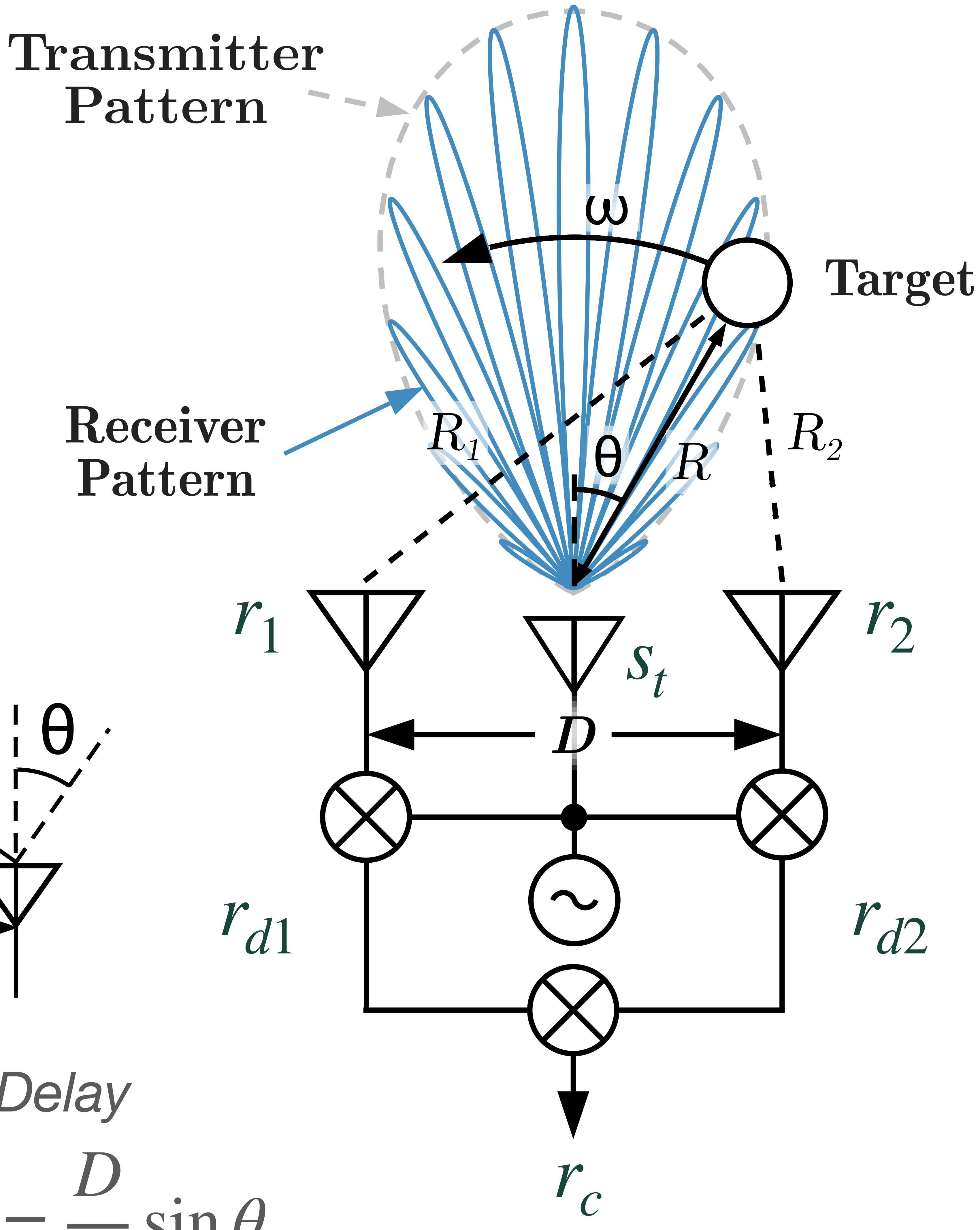
Finally, using $\omega = v/R$

$$v_\theta \approx \frac{f_\omega R}{D_\lambda}$$



Geometric Time Delay

where $\tau_g = \tau_{d2} - \tau_{d1} = \dfrac{D}{c}\sin\theta$



Angle Measurement

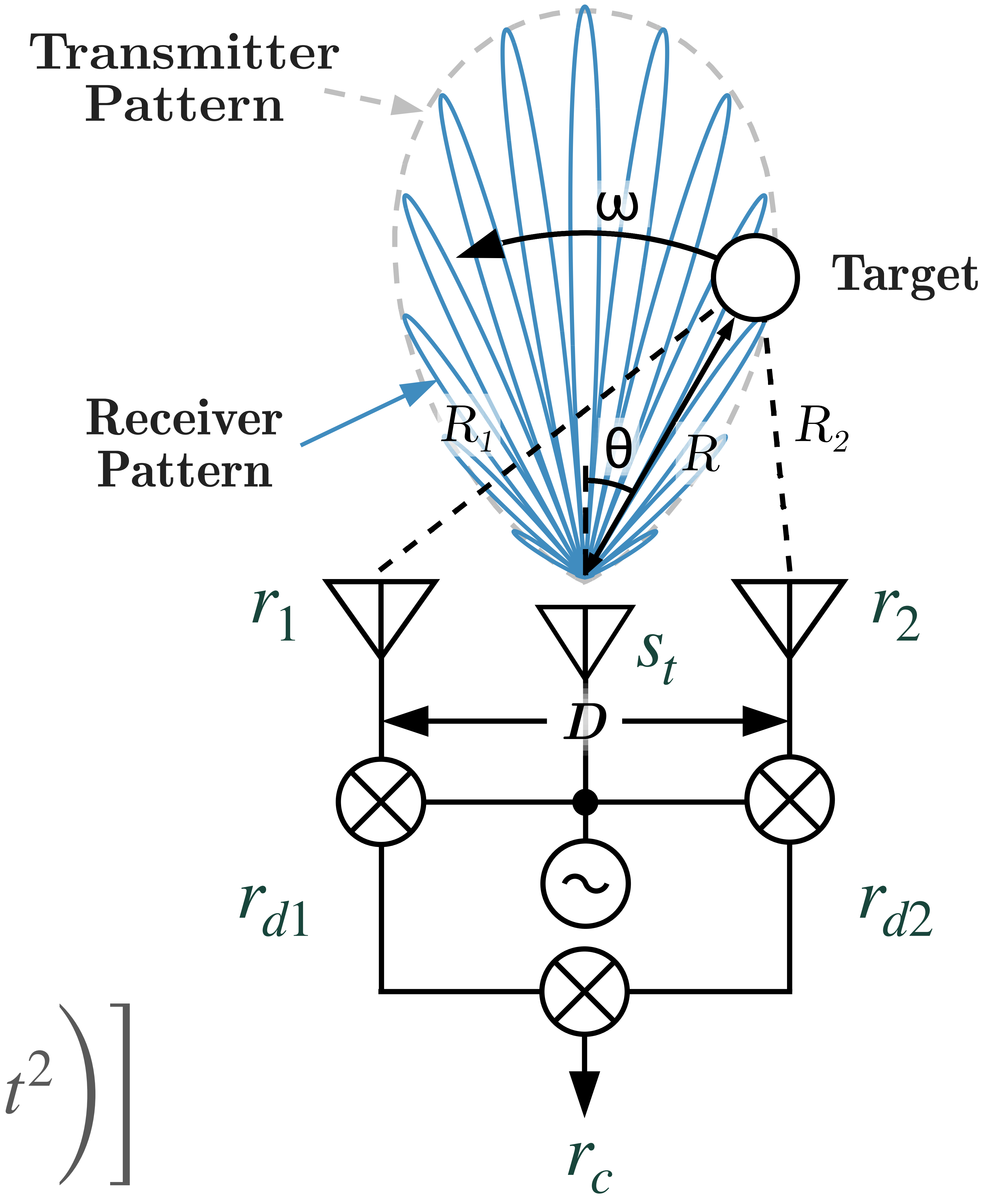
The Interferometric Approach

- To measure absolute angle unambiguously, a modulated waveform is required
- Linear frequency modulation (LFM) will be used in this analysis

$$\omega_t(t) = 2\pi\left(f_0 + Kt\right);\ \ t \in [-\tau/2, \tau/2]$$

where $K = \beta/\tau$ is the chirp rate
β is the chirp bandwidth
τ is the chirp duration

$$s_t(t) = A(\theta)\ \exp\left(j\int \omega_t(t)dt\right) = A(\theta)\ \exp\left[j2\pi\left(f_0 t + \frac{K}{2}t^2\right)\right]$$



Angle Measurement

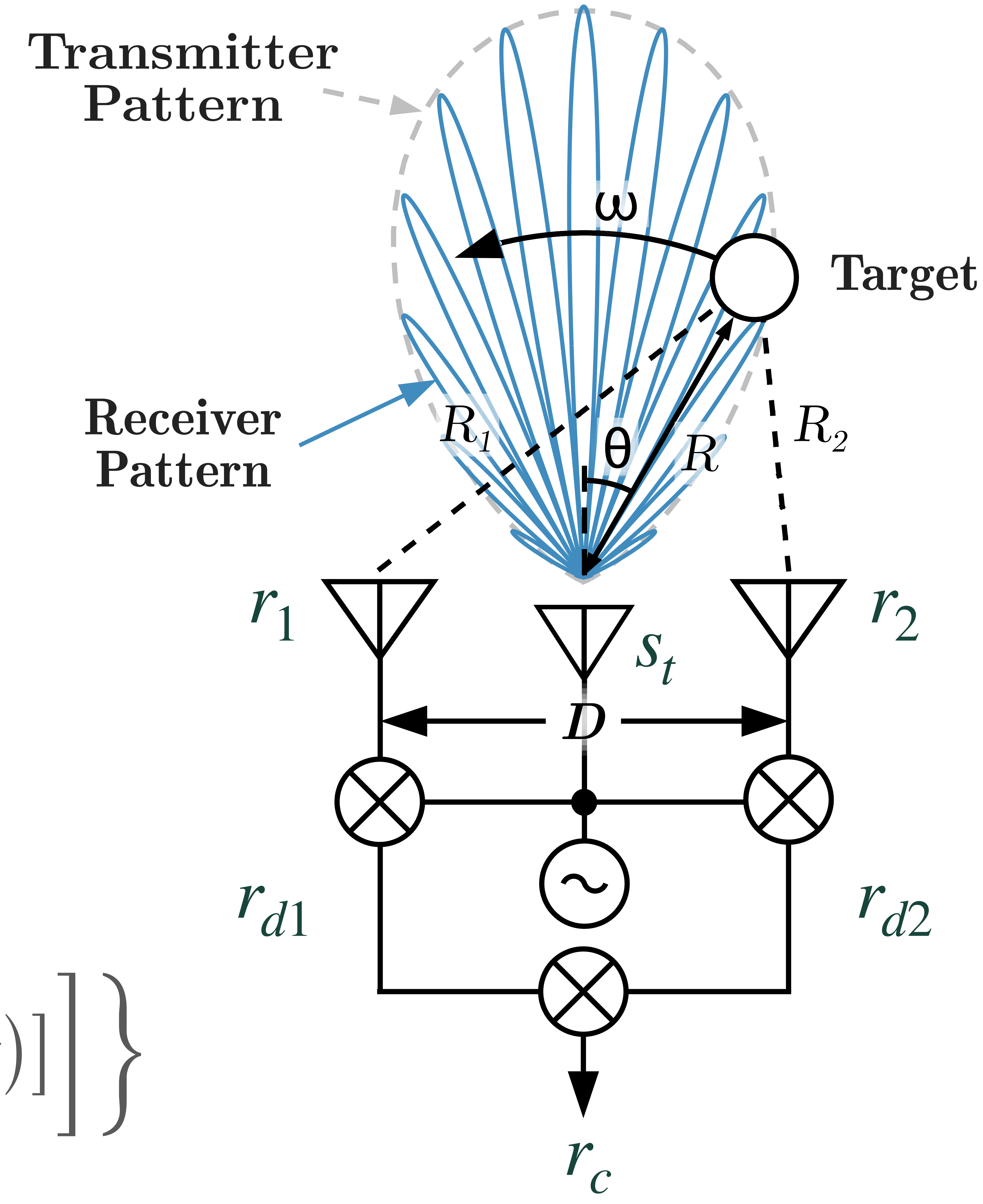
The Interferometric Approach

Downconverted signal at r_{dn}:

$$r_{dn}(t) = r_n(t) \cdot s_t^*(t)$$
$$= A(\theta)\ \exp\left\{ j2\pi \left[-f_0\tau_{dn} + \frac{K}{2}\left(\tau_{dn}^2 - 2\tau_{dn}t\right) \right] \right\}$$

Correlation signal at r_c:

$$r_c(\tau_g, t) = r_1(t) \cdot r_2^*(t)$$
$$= A(\theta)\ \exp\left\{ j2\pi \left[f_0\tau_g + \frac{K}{2}\left[\left(\tau_{d2}^2 - 2\tau_{d2}t\right) - \left(\tau_{d1}^2 - 2\tau_{d1}t\right)\right] \right] \right\}$$



Angle Measurement

The Interferometric Approach

$$r_c(\tau_g, t) = A(\theta)\ \exp\left\{ j2\pi \left[f_0\tau_g + \frac{K}{2}\left[\left(\tau_{d2}^2 - 2\tau_{d2}t\right) - \left(\tau_{d1}^2 - 2\tau_{d1}t\right)\right]\right]\right\}$$

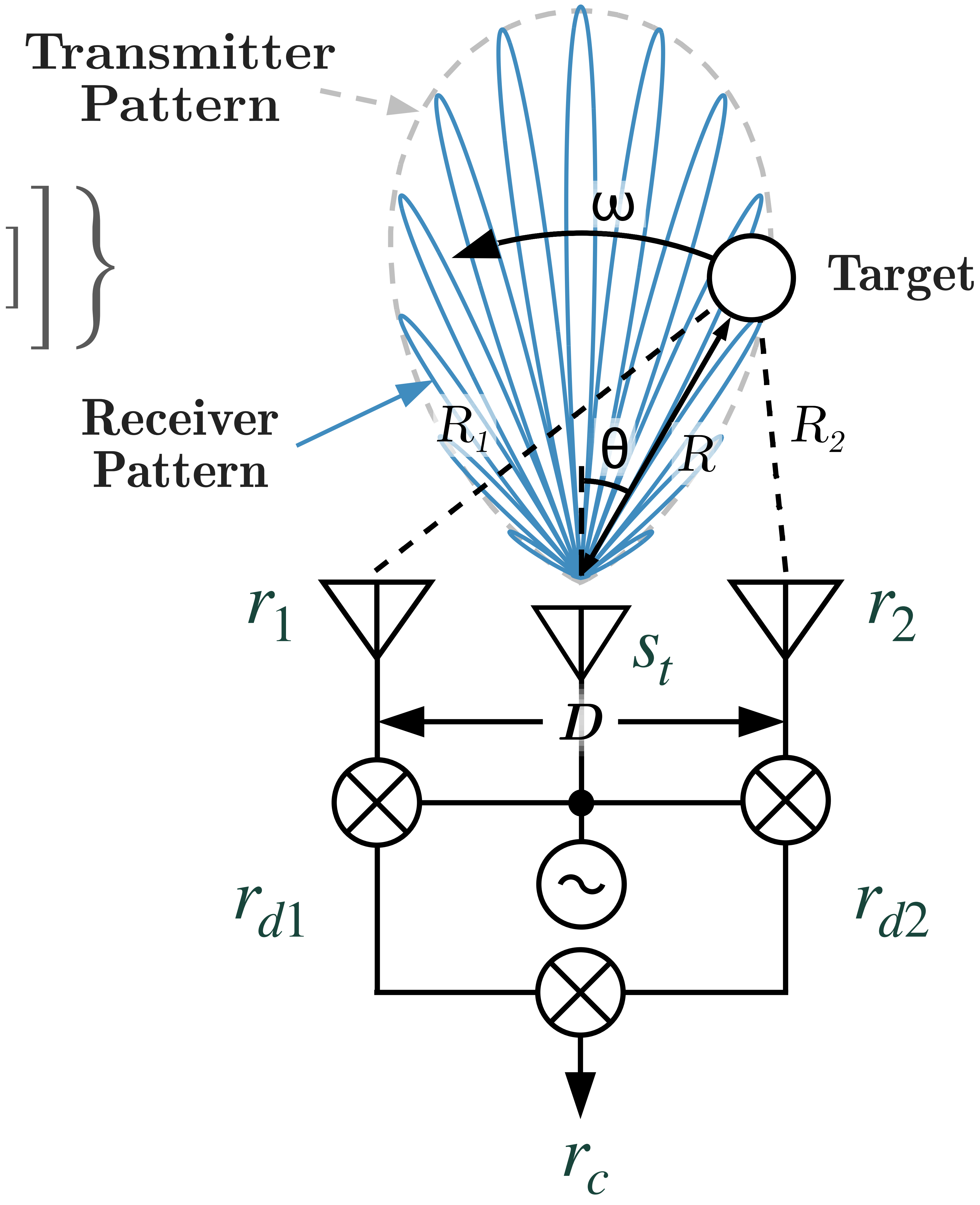
$$= A(\theta)\ \exp\left\{ j2\pi \left[f_0\tau_g + \frac{K}{2}\left(\tau_{d2}^2 - \tau_{d1}^2 - 2\tau_g t\right)\right]\right\}$$

where $\tau_g = \tau_{d2} - \tau_{d1} = \frac{D}{c}\sin\theta$

Angle may be derived from the beat frequency:

$$f_b(\theta, t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

$$= \frac{d}{dt}\left[f_0\tau_g + \frac{K}{2}\left(\tau_{d2}^2 - \tau_{d1}^2 - 2\tau_g t\right)\right]$$



Angle Measurement

The Interferometric Approach

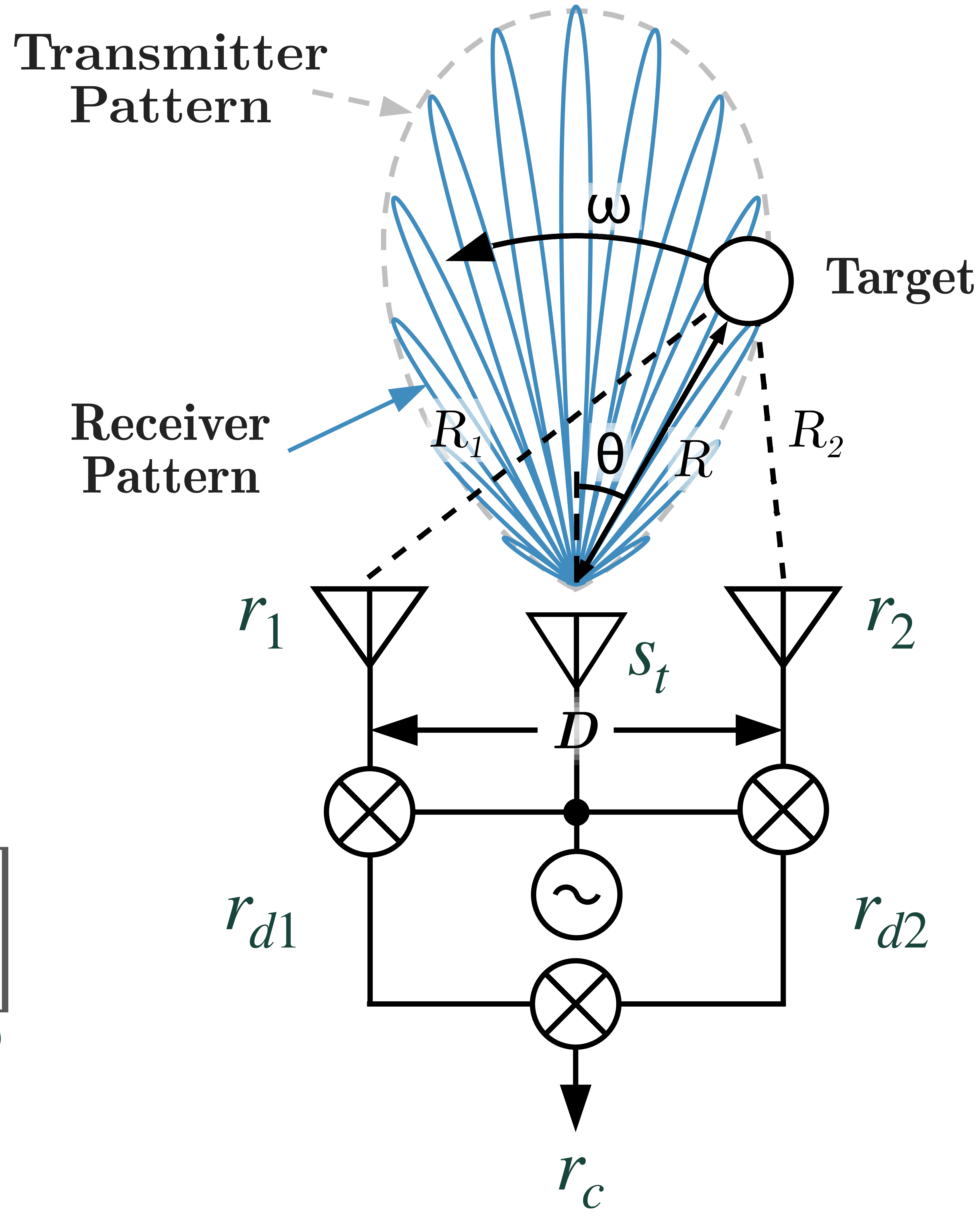
$$f_b(\theta, t) = \frac{d}{dt}\left[f_0\tau_g + \frac{K}{2}\left(\tau_{d2}^2 - \tau_{d1}^2 - 2\tau_g t\right)\right]$$

Note that: $\frac{d\tau_{dn}}{dt} = \frac{2v_{Rn}}{c}$ and $\frac{d\theta}{dt} = \omega \implies \frac{d\tau_g}{dt} = \omega\frac{D}{c}\cos(\theta)$

For a dynamic target $v_{Rn} \neq 0$, the beat frequency is:

$\ll$ first terms

$$f_b(\theta, t) = \underbrace{\omega D_\lambda \cos\theta}_{\text{Angular Velocity}} - K\left[\underbrace{\frac{D}{c}\left(\sin\theta + \omega t\cos\theta\right)}_{\text{Angle}} \underbrace{- \frac{2}{c^2}\left(R_2 v_{R2} - R_1 v_{R1}\right)}_{\text{Intermodulation Terms}}\right]$$



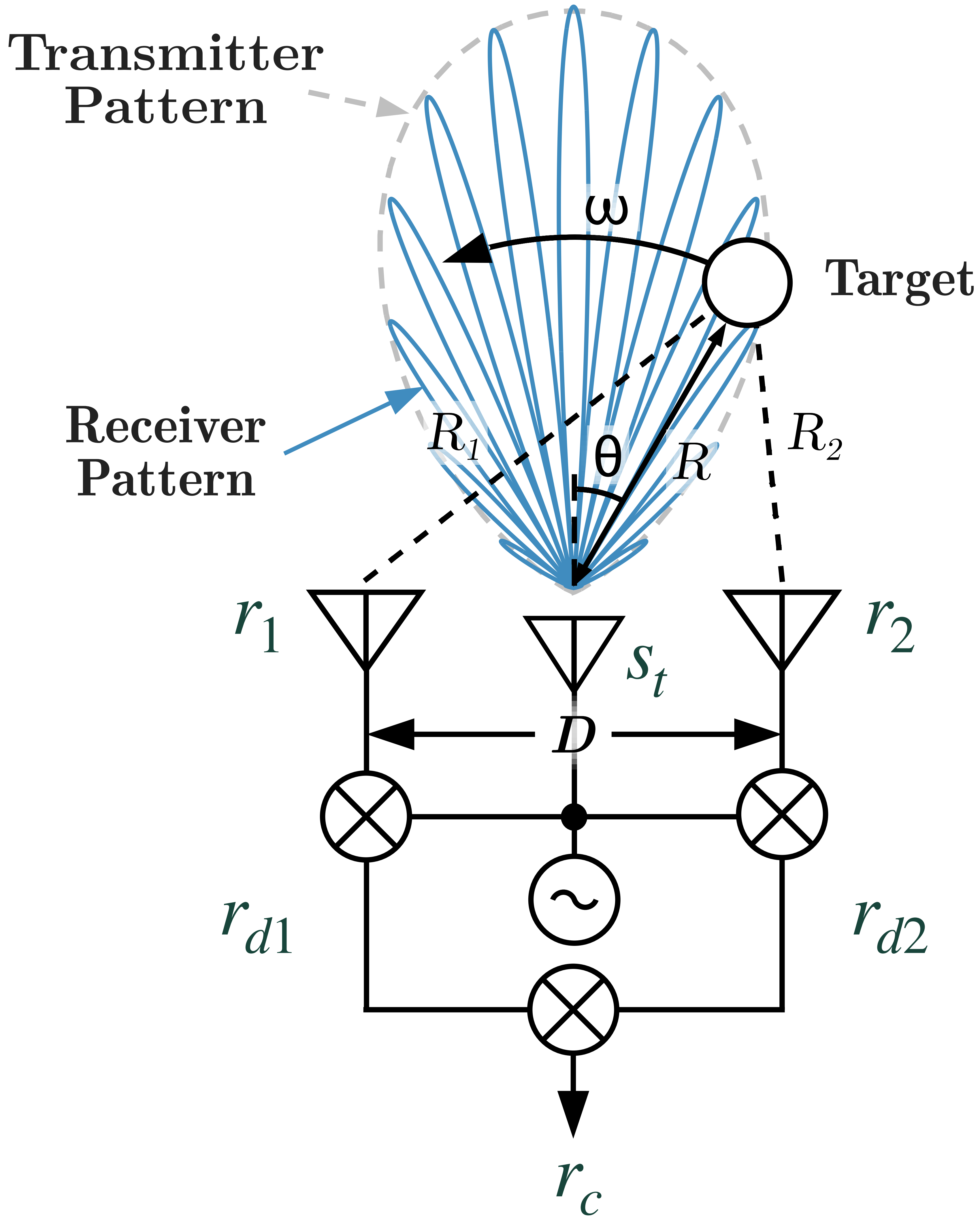
Angle Measurement

The Interferometric Approach

$$f_b(\theta, t) \approx \underbrace{\omega D_\lambda \cos\theta}_{\text{Angular Velocity}} - K\left[\underbrace{\frac{D}{c}\left(\sin\theta}_{\text{Angle}} + \underbrace{\omega t \cos\theta\right)}_{\text{Intermodulation Terms}}\right]$$

Integrating f_b over one chirp:

$$f_b(\theta) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f_b(\theta, t)dt \approx \omega\tau D_\lambda \cos\theta - \beta\frac{D}{c}\sin\theta$$



Angle Measurement

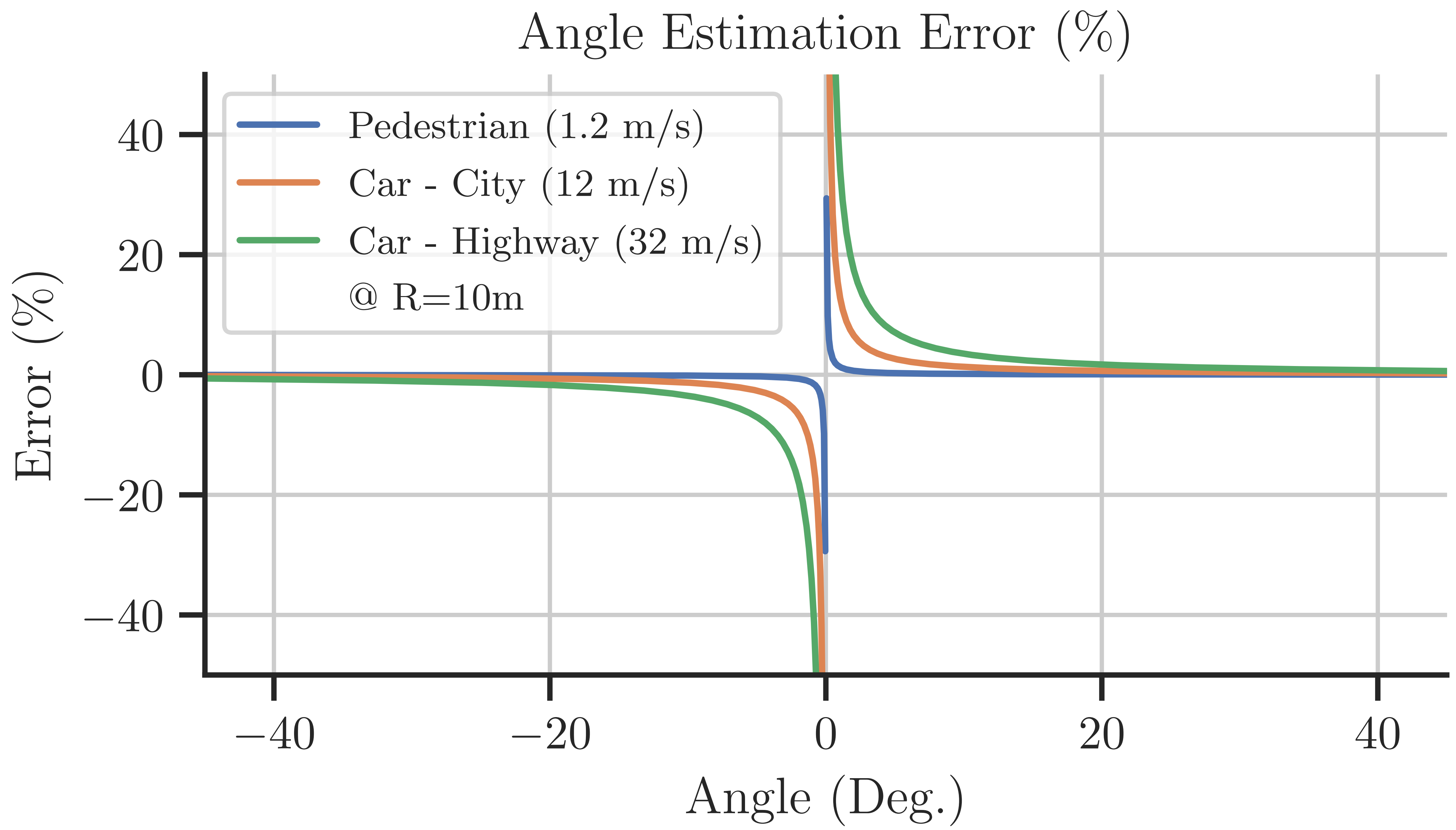
The Interferometric Approach

$$f_b(\theta) \approx \omega\tau D_\lambda \cos\theta - \beta\frac{D}{c}\sin\theta$$

Quasi-static approximation:

$$iff \quad \left|\sin\theta\right| \gg \left|\omega\frac{f_0}{K}\cos\theta\right|$$

then $$\boxed{\theta \approx \sin^{-1}\left(-\frac{c}{\beta D}f_b\right)}$$



Angle percent error using quasi-static approximation for various targets at 10m radius

Angular Rate Measurement (LFM)

The Interferometric Approach

Angular rate is derived from the phase at the correlator, ϕ_c

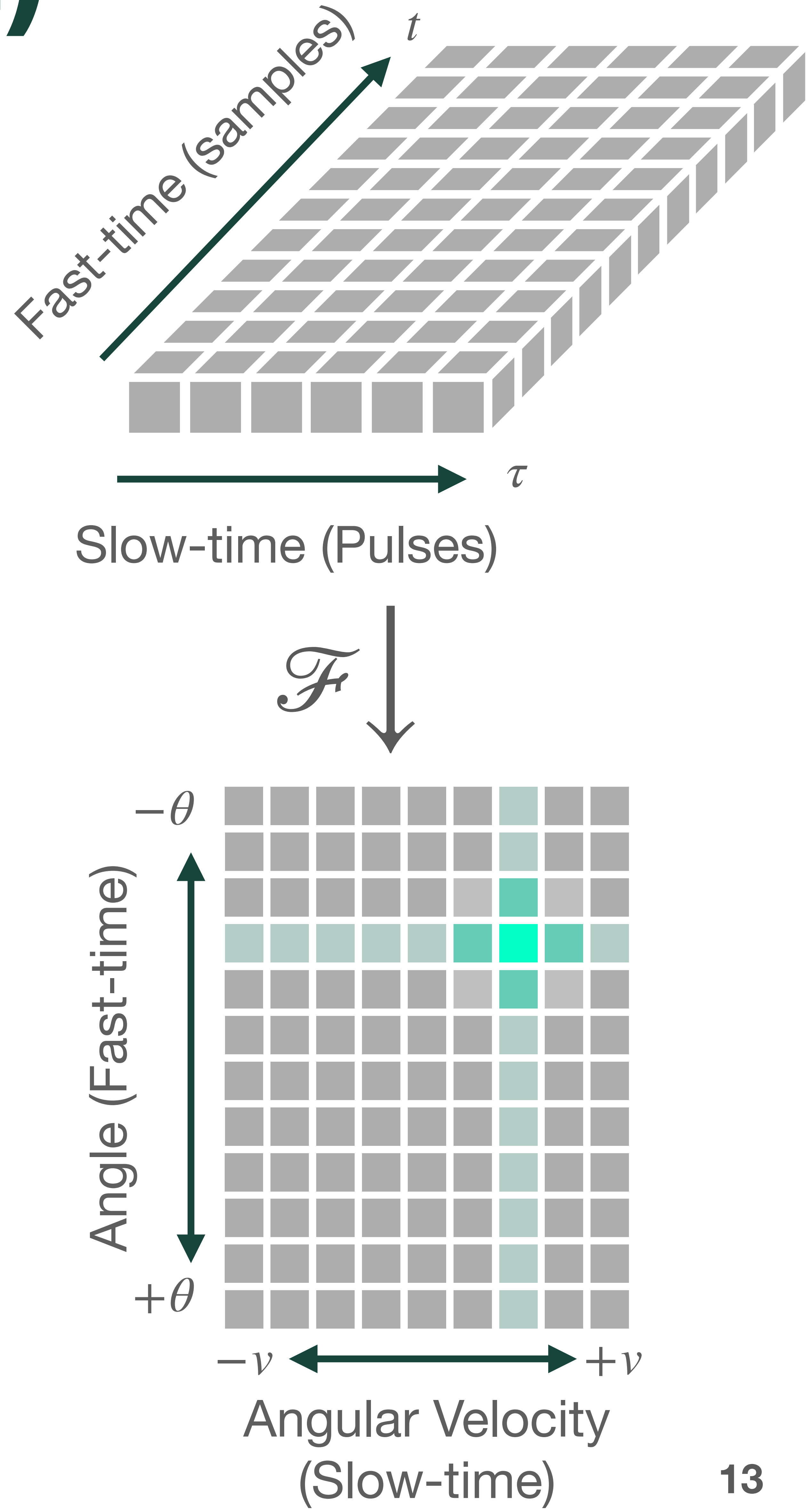
$$\phi_c(n) = 2\pi \left[f_0 \tau_g(n) + \frac{K}{2} \left(\tau_{d2}^2(n) - \tau_{d1}^2(n) - 2\tau_g(n)t \right) \right]$$

Recall $\tau_g(n) = \dfrac{D}{c} \sin\theta(n)$ where n is the number of pulses

$$f_\omega = \frac{1}{2\pi}\frac{d\phi_c}{dn} = \omega \frac{D}{c}\cos\theta \left(f_0 - \frac{\beta}{\tau} t \right); \quad t \in \left[-\frac{\tau}{2}, \frac{\tau}{2} \right]$$

$$\textit{iff} \quad f_0 \gg \frac{\beta}{2} \implies \omega \approx f_\omega \frac{1}{D_\lambda \cos\theta}$$

or $\boxed{v_\theta \approx f_\omega \dfrac{R}{D_\lambda}}$ for small angles, and $\boxed{\theta \approx \sin^{-1}\left(-\dfrac{c}{\beta D} f_\theta \right)}$



Validation of Theory

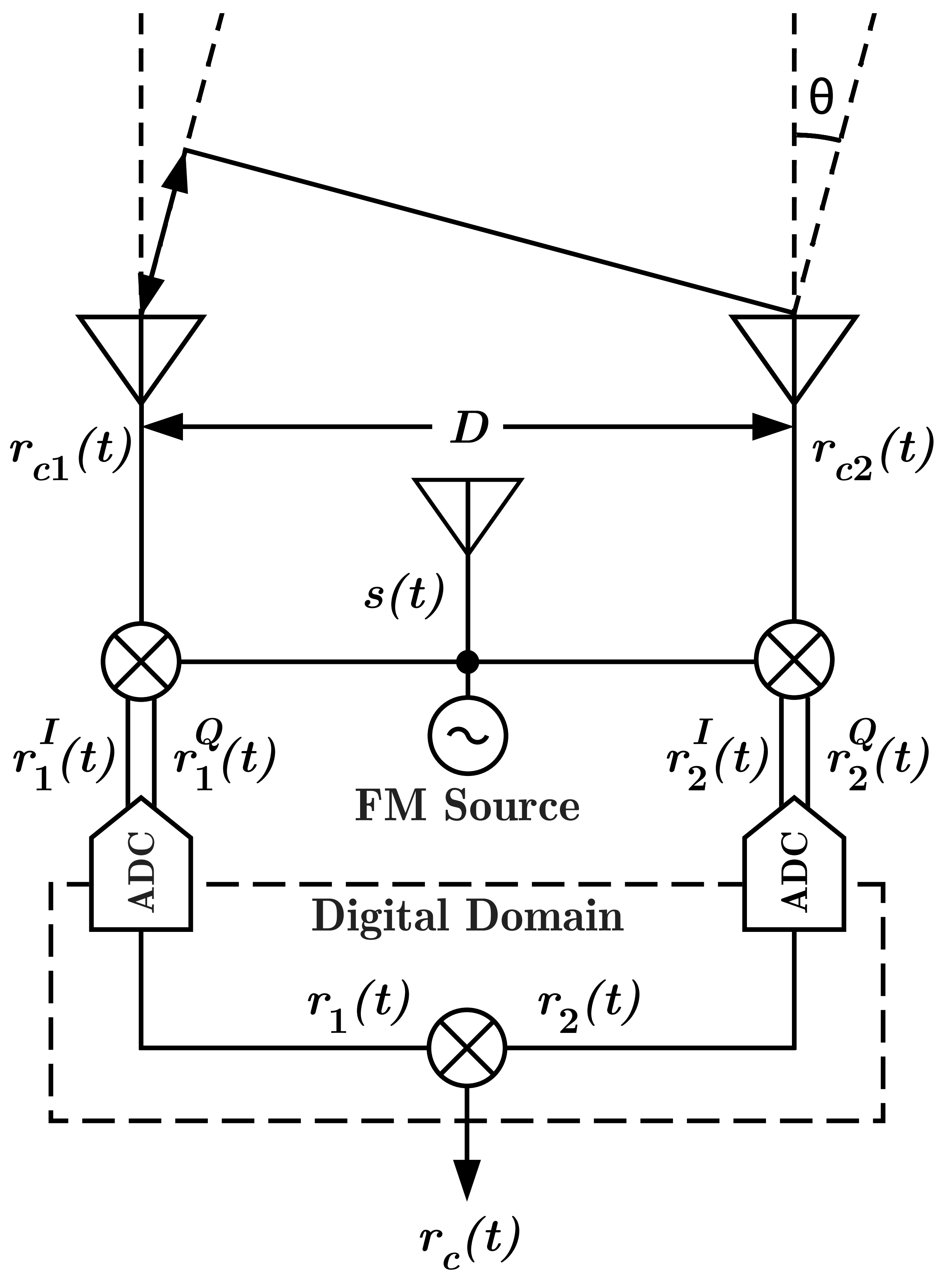
Simulated Linear Frequency Modulated Interferometer

Proposed Active LFM Interferometer

Validation of Theory - Linear Frequency Modulated Interferometer

- Utilizes conventional, low-cost heterodyning architecture
- Correlation occurs in the digital domain
- Governed by simple fundamental parameter relations:

Antenna baseline, D	$f_\omega, f_\theta \propto D$
Carrier wavelength, λ	$f_\omega, f_D \propto 1/\lambda$
Chirp-rate, K	$f_\theta, f_R \propto K$

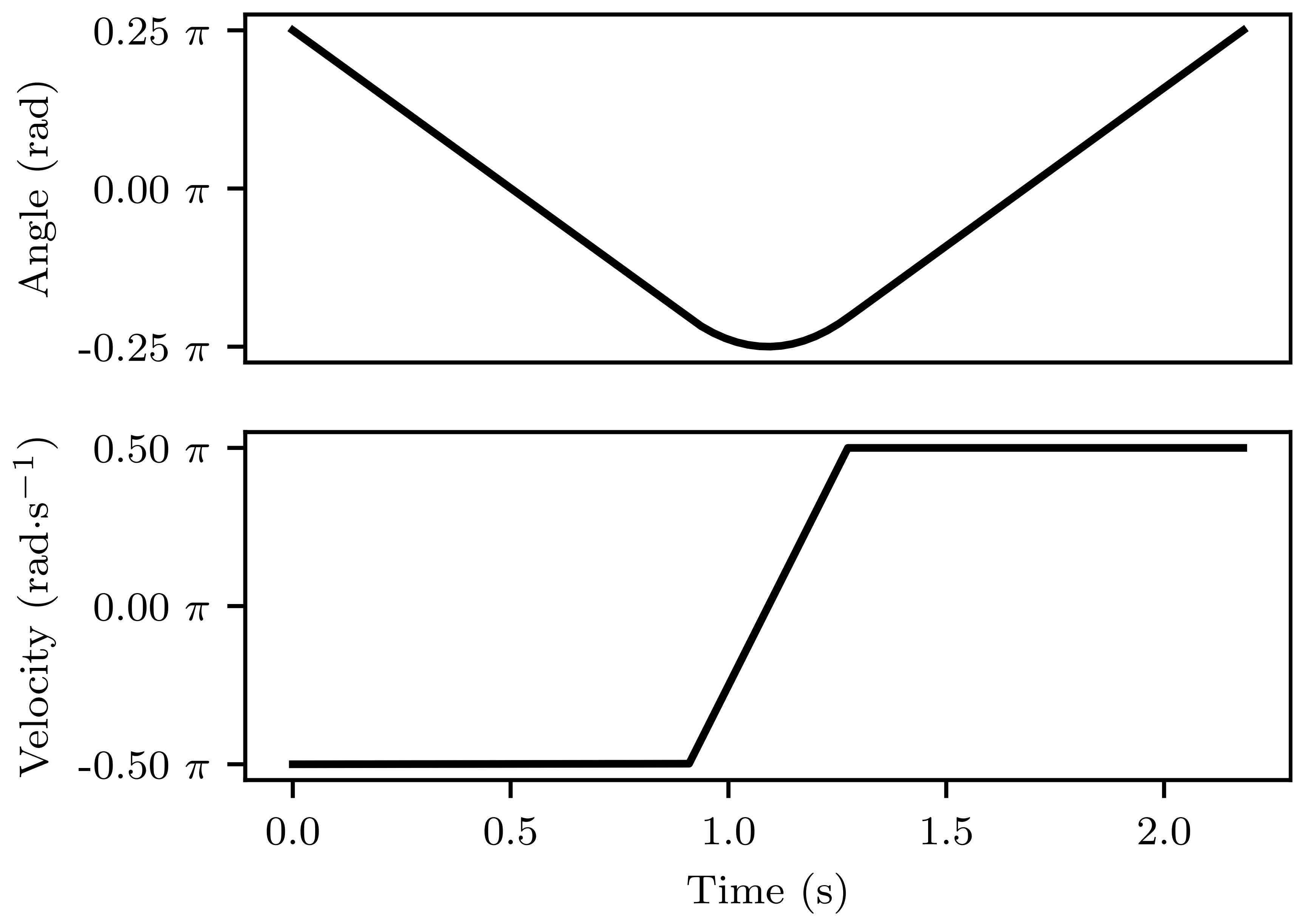


Proposed LFM Interferometer

Simulation Configuration

Validation of Theory - Linear Frequency Modulated Interferometer

- Initial validation performed using simulation
- Simulation parameters:
  - $f_0 \in \{5.8, 11.6\}$ GHz, $\beta = 100$ MHz, $\tau \in \{100, 200\}$ μs
  - $D \in \{10, 20\} \cdot \lambda$, $R = 10$ m
  - $|\omega| = \pi/2 \text{ rad} \cdot \text{s}^{-1}$, $|\gamma| = 5\pi/2 \text{ rad} \cdot \text{s}^{-2}$
  - Parameters chosen to be replicate able with physical hardware

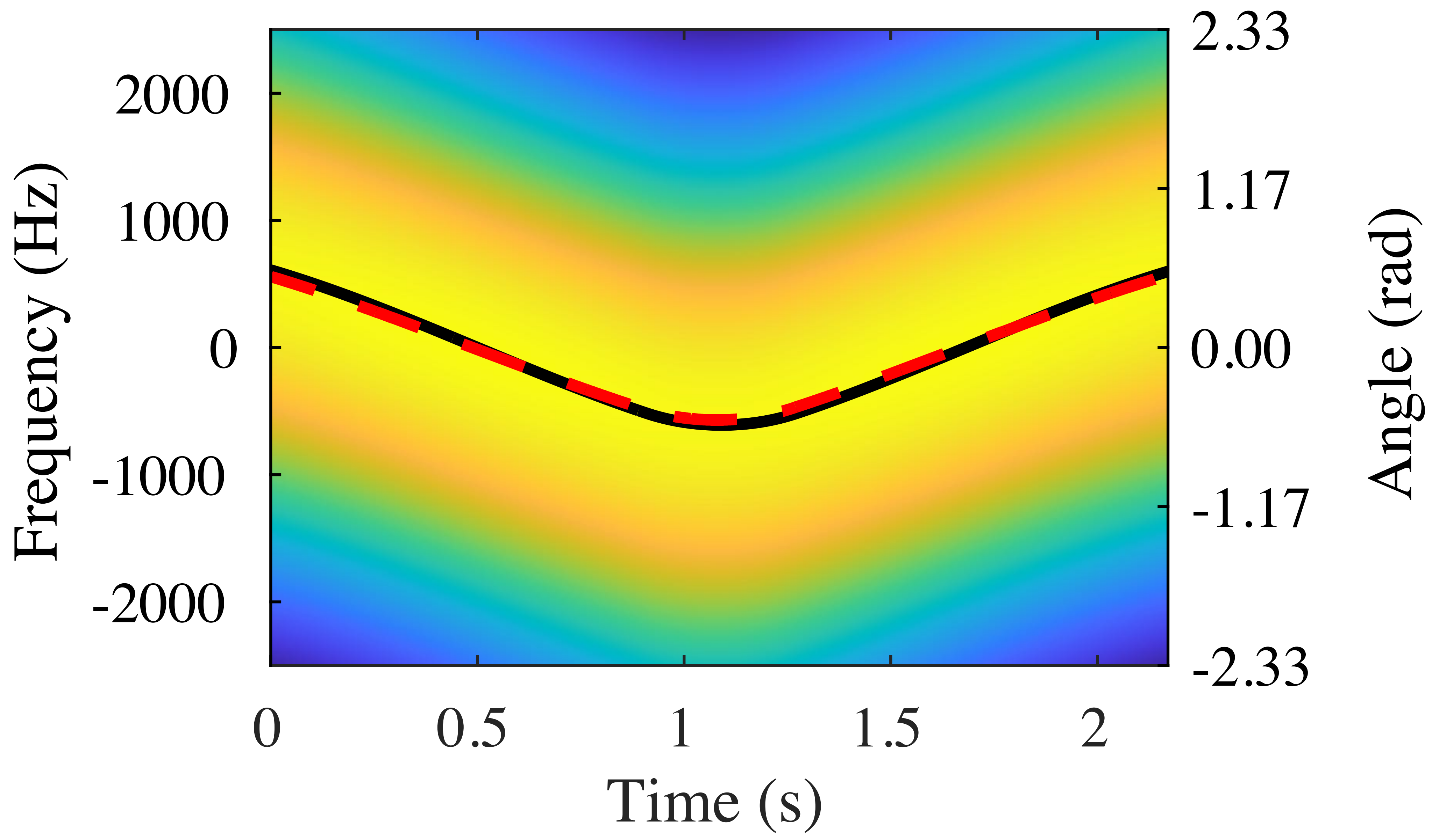


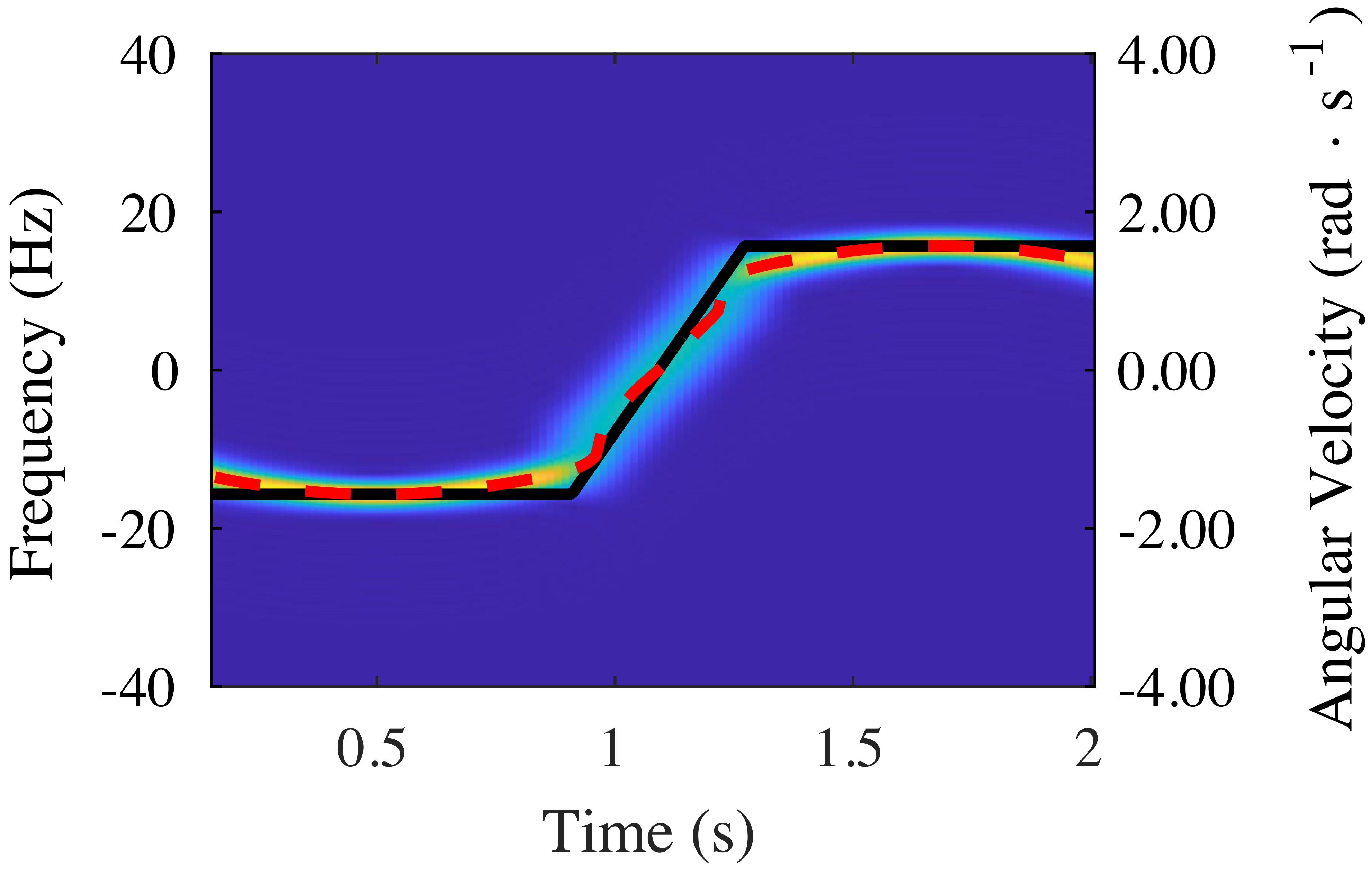
Simulated target trajectory

Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

$$f_0 = 5.8\,\text{GHz},\ \beta = 100\,\text{MHz},\ \tau = 200\,\mu\text{s},\ D = 10 \cdot \lambda$$



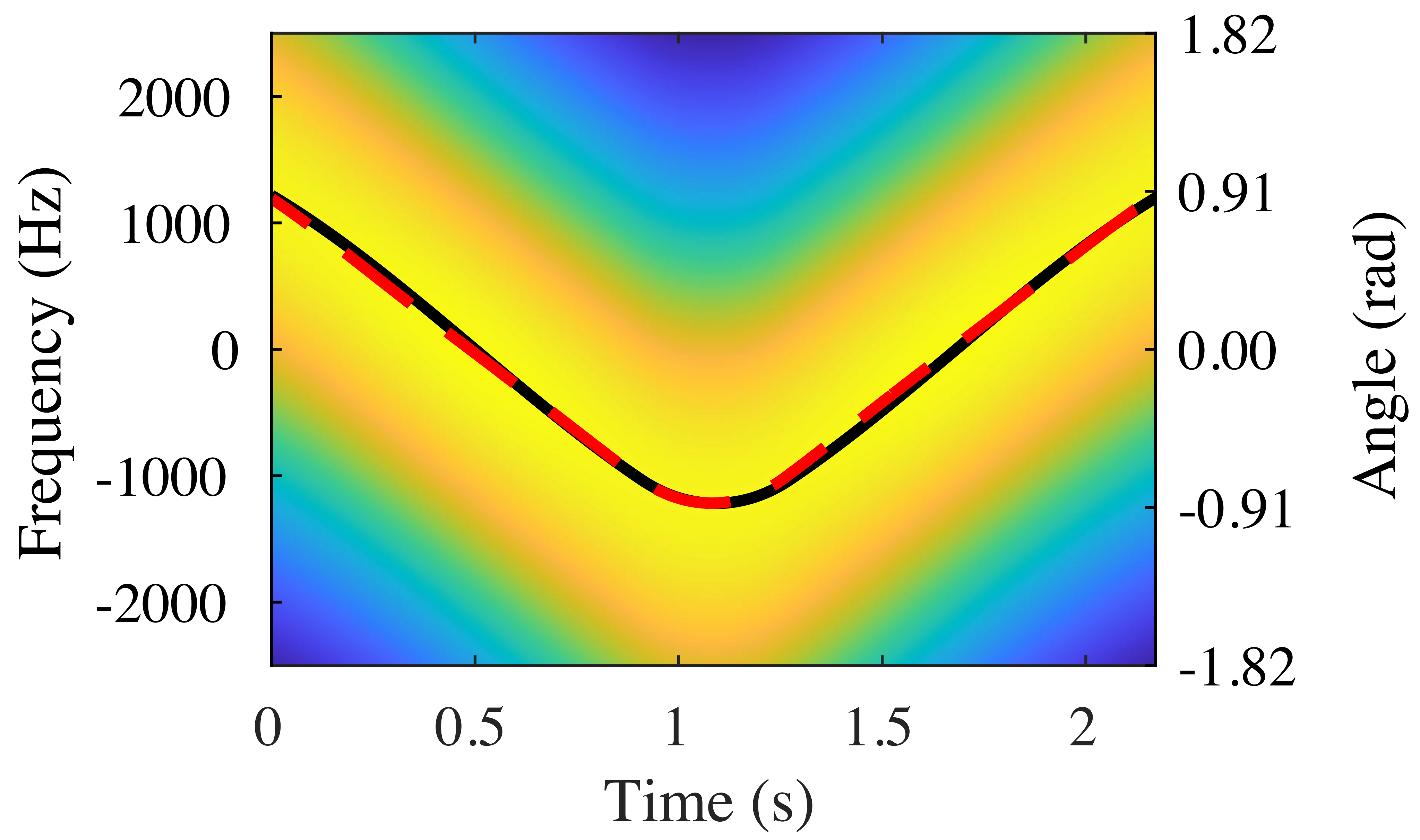


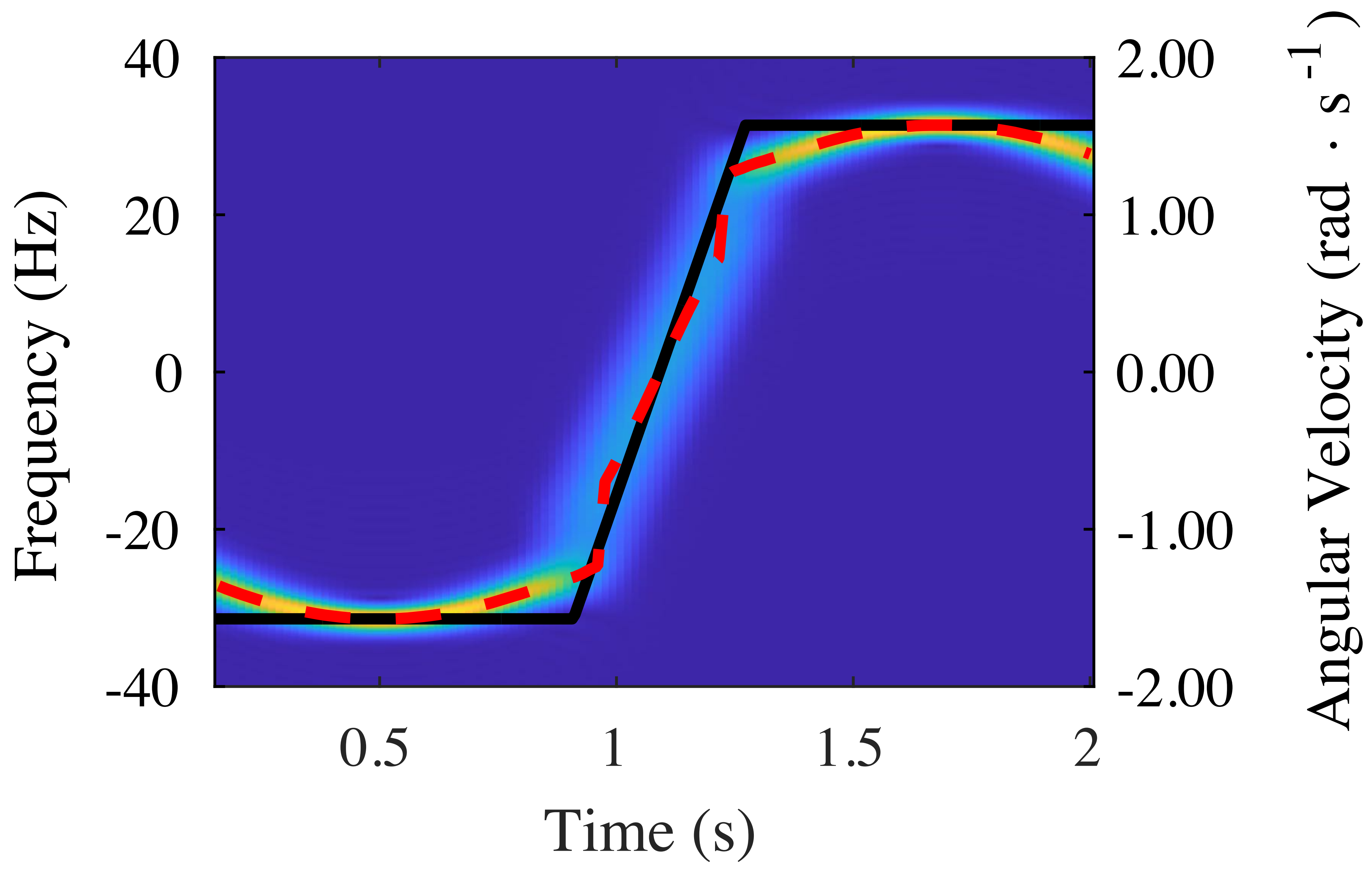
Angle RMSE: 0.0377 rad *Ang. Vel. RMSE:* $0.1784\ \text{rad} \cdot \text{s}^{-1}$

Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

$f_0 = 5.8\,\text{GHz},\ \beta = 100\,\text{MHz},\ \tau = 200\,\mu\text{s},\ D = 20 \cdot \lambda$



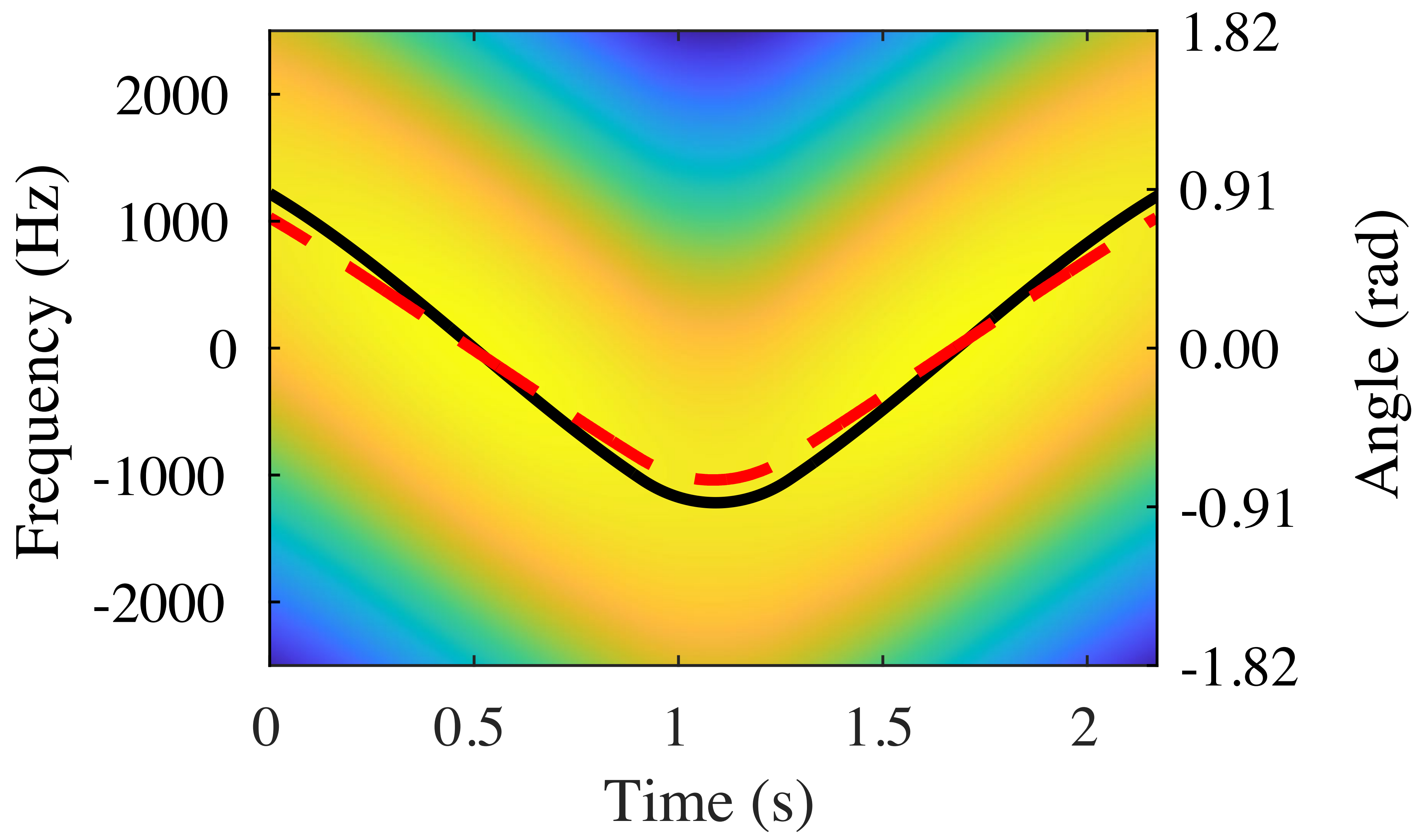


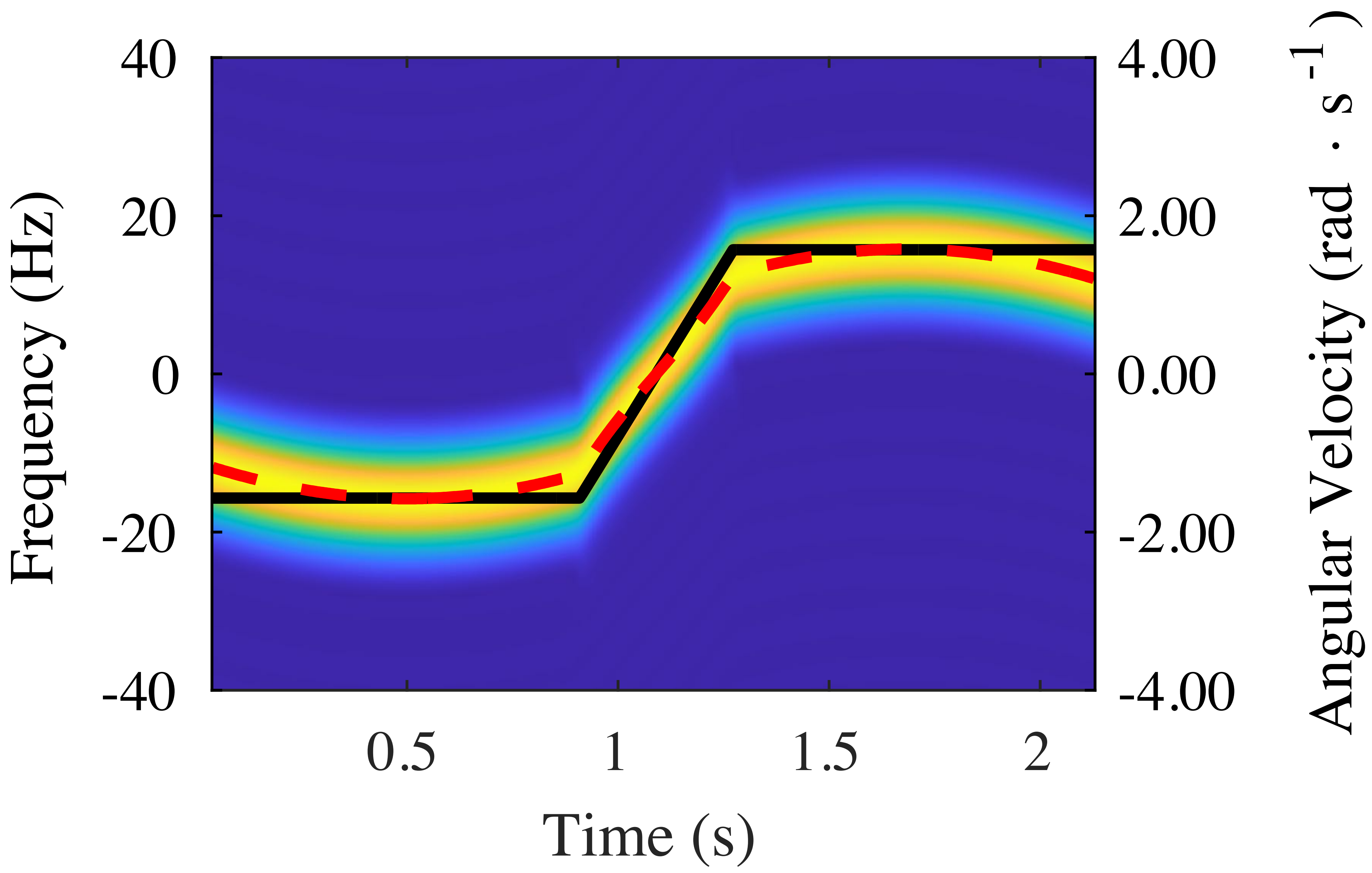
Angle RMSE: $0.0224\ \text{rad}$ *Ang. Vel. RMSE:* $0.1613\ \text{rad} \cdot \text{s}^{-1}$

Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

$$f_0 = 5.8\,\text{GHz},\ \beta = 100\,\text{MHz},\ \tau = 100\,\mu\text{s},\ D = 10 \cdot \lambda$$



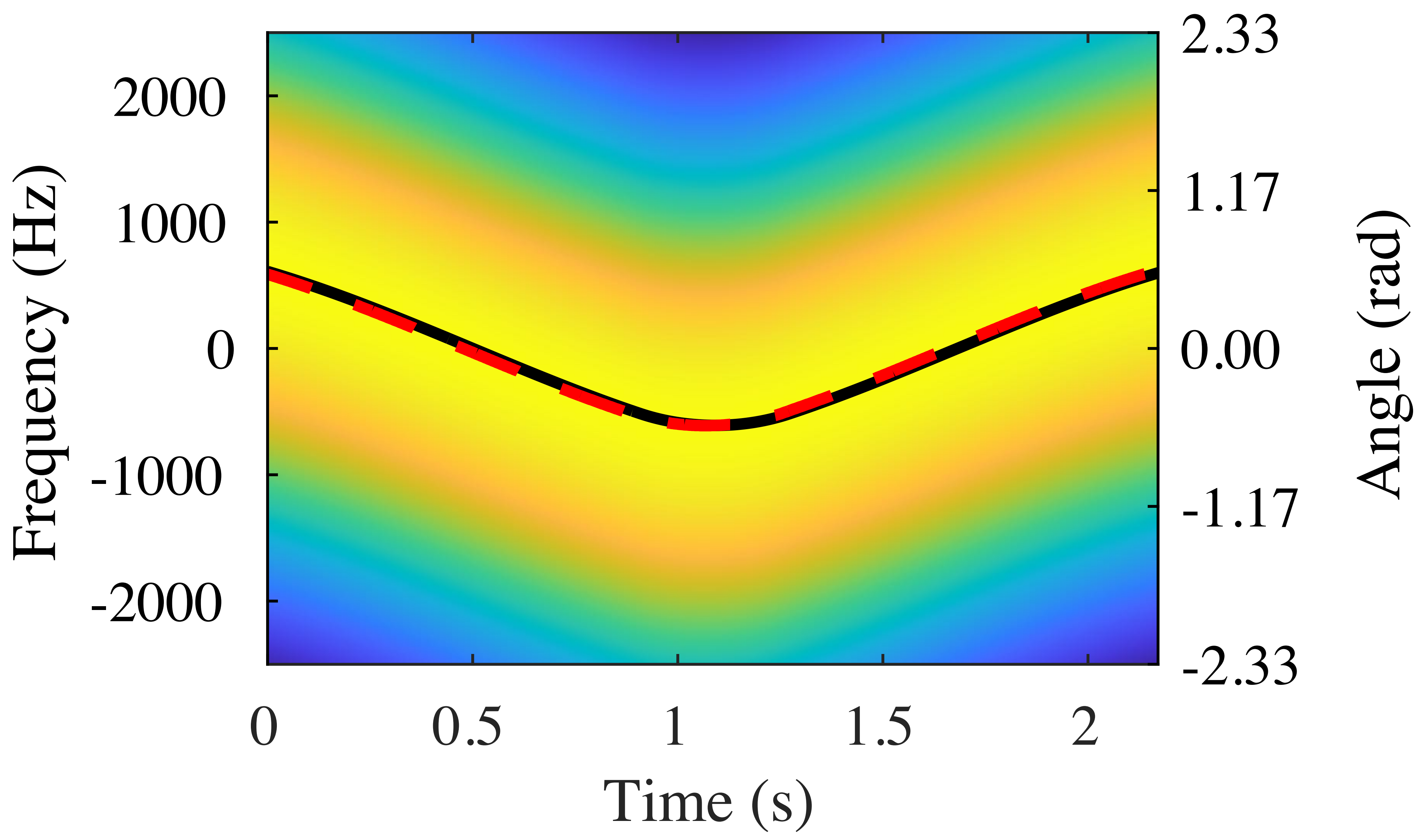


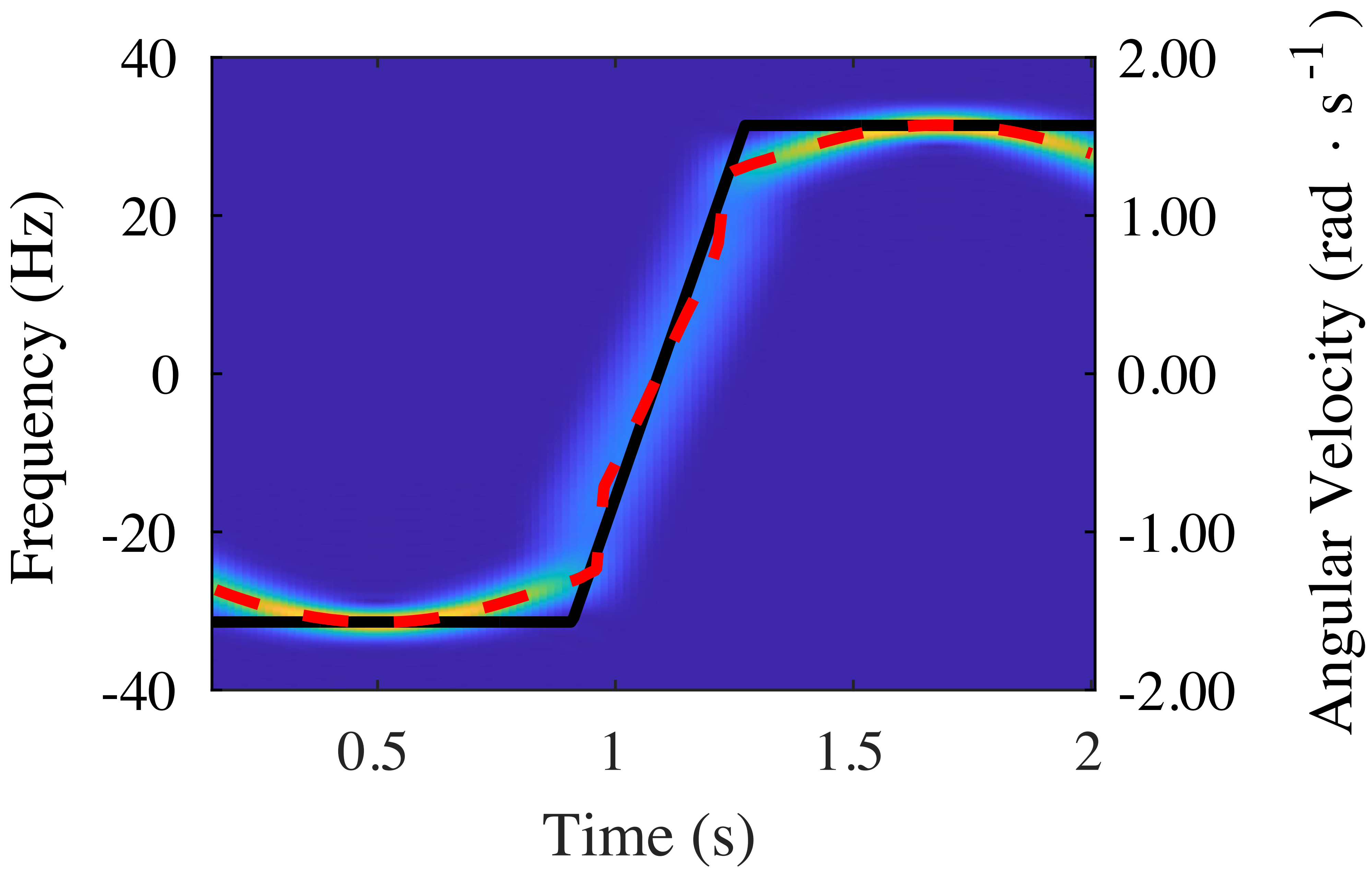
Angle RMSE: 0.0767 rad *Ang. Vel. RMSE:* $0.1760\ \text{rad} \cdot \text{s}^{-1}$

Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

$f_0 = 11.6\,\text{GHz}$, $\beta = 100\,\text{MHz}$, $\tau = 200\,\mu\text{s}$, $D^* = 20 \cdot \lambda$





Angle RMSE: 0.0316 rad   *Ang. Vel. RMSE:* $0.1550\ \text{rad} \cdot \text{s}^{-1}$

Simulation Results

Validation of Theory - Linear Frequency Modulated Interferometer

$f_0 = 5.8\,\mathrm{GHz}$, $\beta = 100\,\mathrm{MHz}$, $\tau = 200\,\mu\mathrm{s}$, $D = 10 \cdot \lambda$

Angle RMSE: 0.0377 rad
Ang. Vel. RMSE: $0.1784\ \mathrm{rad} \cdot \mathrm{s}^{-1}$

$f_0 = 5.8\,\mathrm{GHz}$, $\beta = 100\,\mathrm{MHz}$, $\tau = 200\,\mu\mathrm{s}$, $D = 20 \cdot \lambda$

Angle RMSE: 0.0224 rad
Ang. Vel. RMSE: $0.1613\ \mathrm{rad} \cdot \mathrm{s}^{-1}$

$f_0 = 5.8\,\mathrm{GHz}$, $\beta = 100\,\mathrm{MHz}$, $\tau = 100\,\mu\mathrm{s}$, $D = 10 \cdot \lambda$

Angle RMSE: 0.0767 rad
Ang. Vel. RMSE: $0.1760\ \mathrm{rad} \cdot \mathrm{s}^{-1}$

$f_0 = 11.6\,\mathrm{GHz}$, $\beta = 100\,\mathrm{MHz}$, $\tau = 200\,\mu\mathrm{s}$, $D^* = 20 \cdot \lambda$

Angle RMSE: 0.0316 rad
Ang. Vel. RMSE: $0.1550\ \mathrm{rad} \cdot \mathrm{s}^{-1}$

Conclusions

- New radar architecture and equation derivations for:
  - Direct, simultaneous measurement of **angle and angular velocity** using an **LFM waveform** and **correlation interferometry** for a point-target
  - Less complex than a dense, beamforming arrays typically used for angle estimation

- Simulated validation of:
  - Simultaneous **direct measurement of angle of arrival and angular velocity** of a point-target using LFM waveform using a simple process **analogous to range-Doppler processing**

Questions?

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Backup Slides

Six Degree of Freedom Measurements

The Interferometric Approach

Position

$$R = \|P\| \approx -f_{bn}\frac{c}{2K}$$

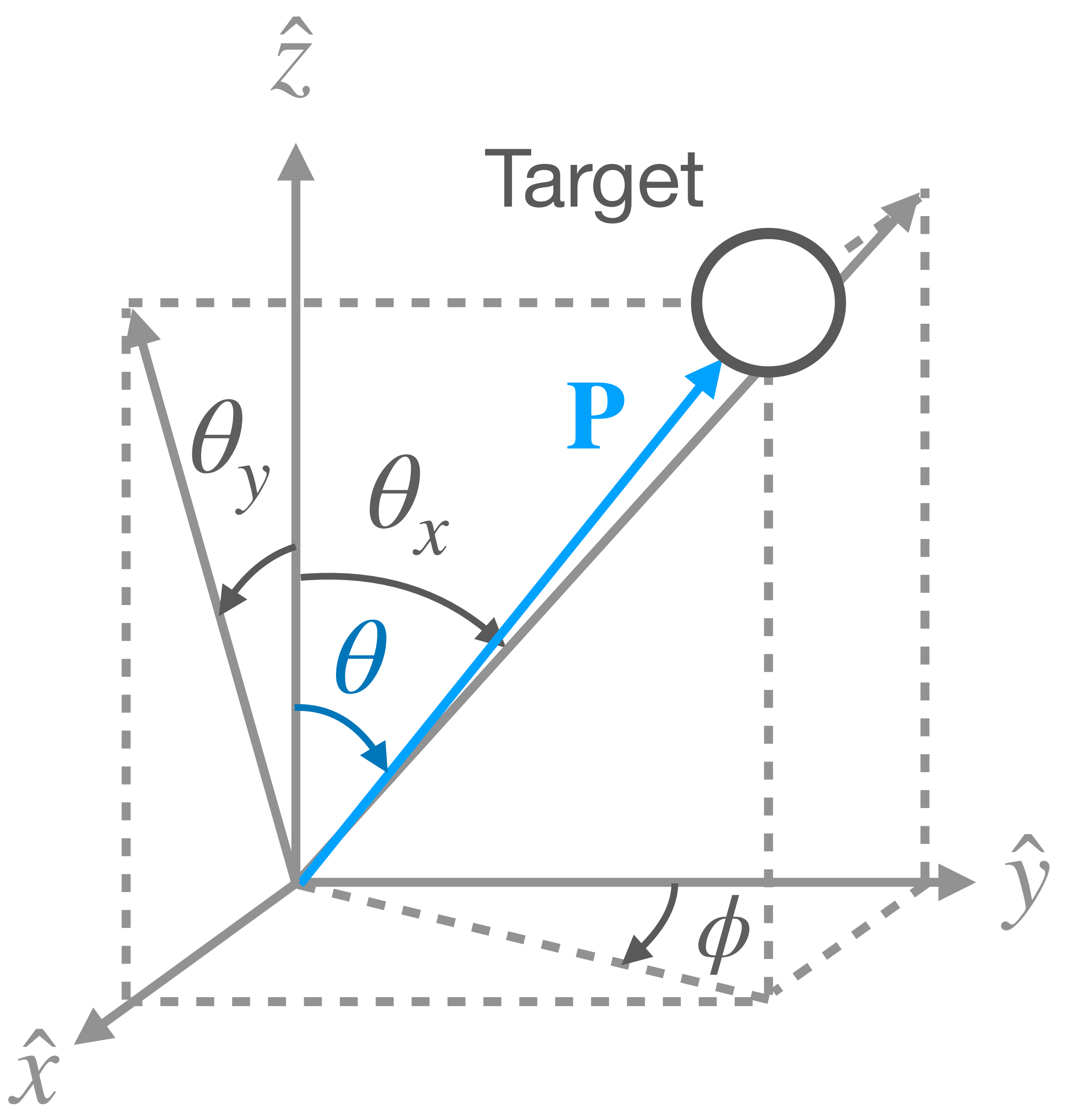
$$\theta_i \approx \sin^{-1}\left(-\frac{\tau}{\beta}\frac{c}{D}f_s\right);\ i \in \{x, y\}$$

} *Directly measured*

$$\theta = \operatorname{atan2}\left(\tan\theta_x,\ \sin\phi\right)$$ *or* $$\theta = \operatorname{atan2}\left(\tan\theta_y,\ \cos\phi\right)$$

$$\phi = \operatorname{atan2}\left(F(\theta_y), F(\theta_x)\right);\ \theta_i \in [-\pi/2, \pi/2]$$

where $$F(\alpha) = \frac{R\tan\alpha}{\sqrt{\tan^2\theta_x\tan^2\theta_y + \tan^2\theta_x + \tan^2\theta_y}}$$



Positional Coordinate System

Six Degree of Freedom Measurements

The Interferometric Approach

Velocity

$$v_R = -\frac{f_d \lambda}{2\tau}$$

$$v_{\theta_i} = f_\omega \frac{R}{D_\lambda \tau}; \; i \in \{x, y\}$$

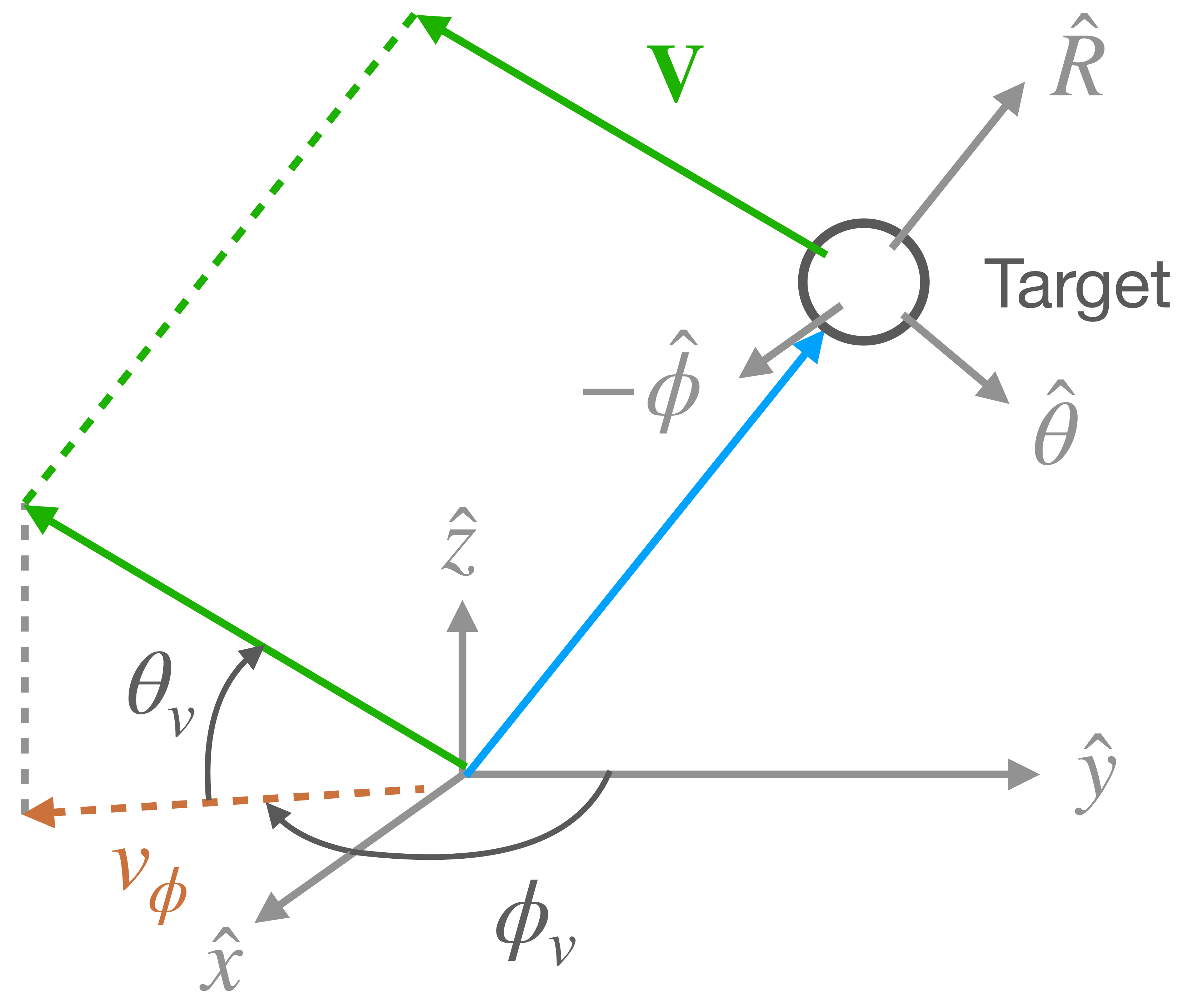
Directly measured

$$\|\mathbf{V}\| = \sqrt{v_R^2 + v_{\theta_x}^2 + v_{\theta_y}^2}$$

$$\theta_v = \text{atan2}\left(v_\phi, v_R\right)$$

$$v_\phi = \sqrt{v_{\theta_x}^2 + v_{\theta_y}^2}$$

$$\phi_v = \text{atan2}\left(v_{\theta_x}, v_{\theta_y}\right)$$



Velocity Coordinate System

Current Methods

State of the Art

Current Methods

Current radars **only perform direct estimates** of:

- **Range**
  - Phase interferometry, waveform modulation (AM, FM, PM)
- **Range-rate**
  - Doppler shift
- **Angle**
  - Mechanical scanning, amplitude comparison, FDoA, TDoA, beamforming, correlative interferometry

Modern radars apply a **locate and track** method **to derive angular-rate**

Radial Velocity Measurement

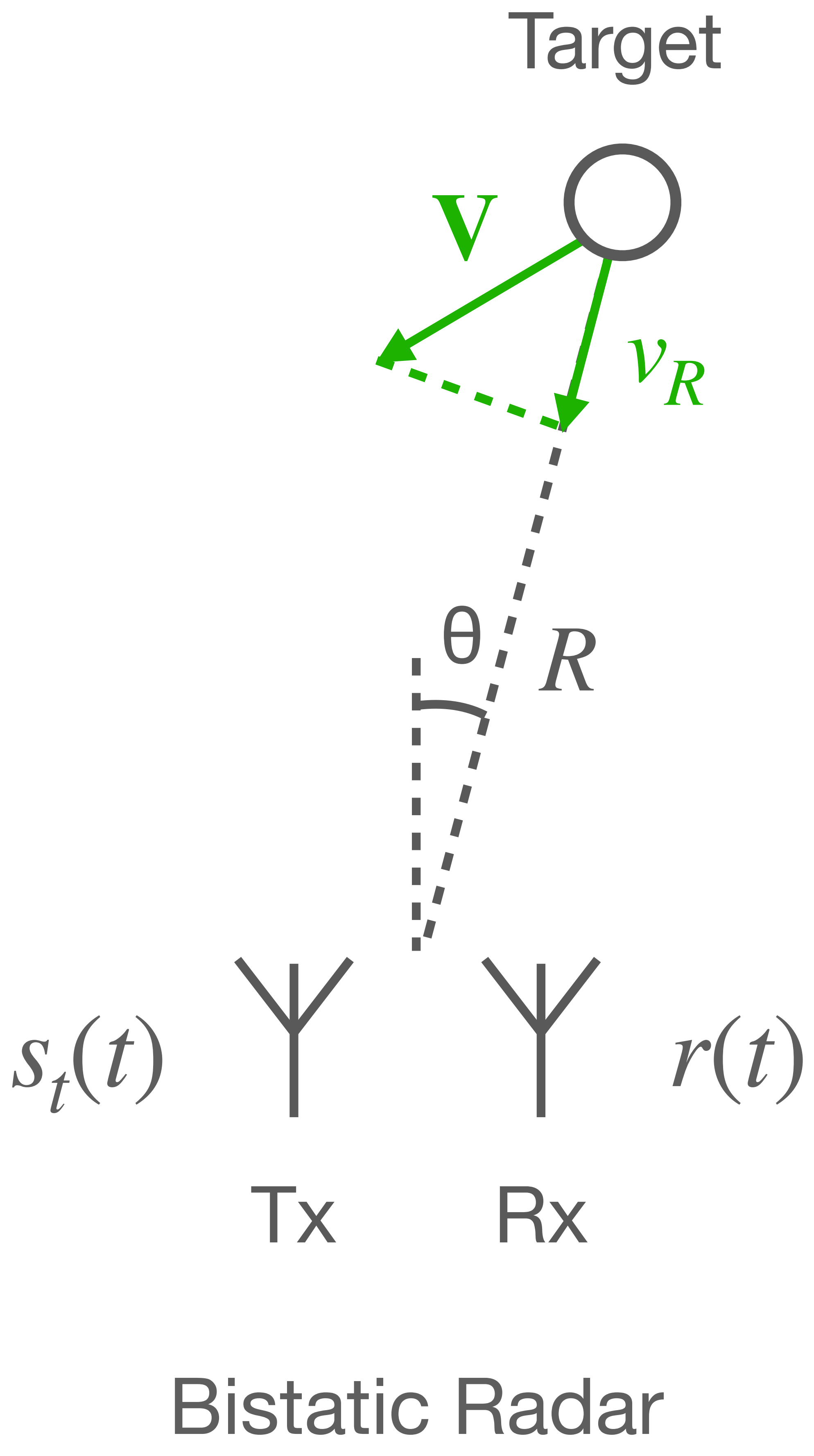
Current Methods

- Doppler is used for direct velocity measurement based on frequency shift
- Can be employed for continuous-wave or modulated waveforms with periodic, stationary phase points

$$s_t(t) = \exp\left(j2\pi f_0 t\right) \qquad r(t) = \exp\left[j2\pi f_0\left(t - \tau_d\right)\right]$$

where $\tau_d = \dfrac{2R}{c}$

$$r_d(t) = r(t) \cdot s_t^*(t)$$
$$= \exp\left(-j2\pi f_0 \tau_d\right)$$



Radial Velocity Measurement

Current Methods

$$r_d(t) = \exp\left(-j2\pi f_0 \tau_d\right)$$

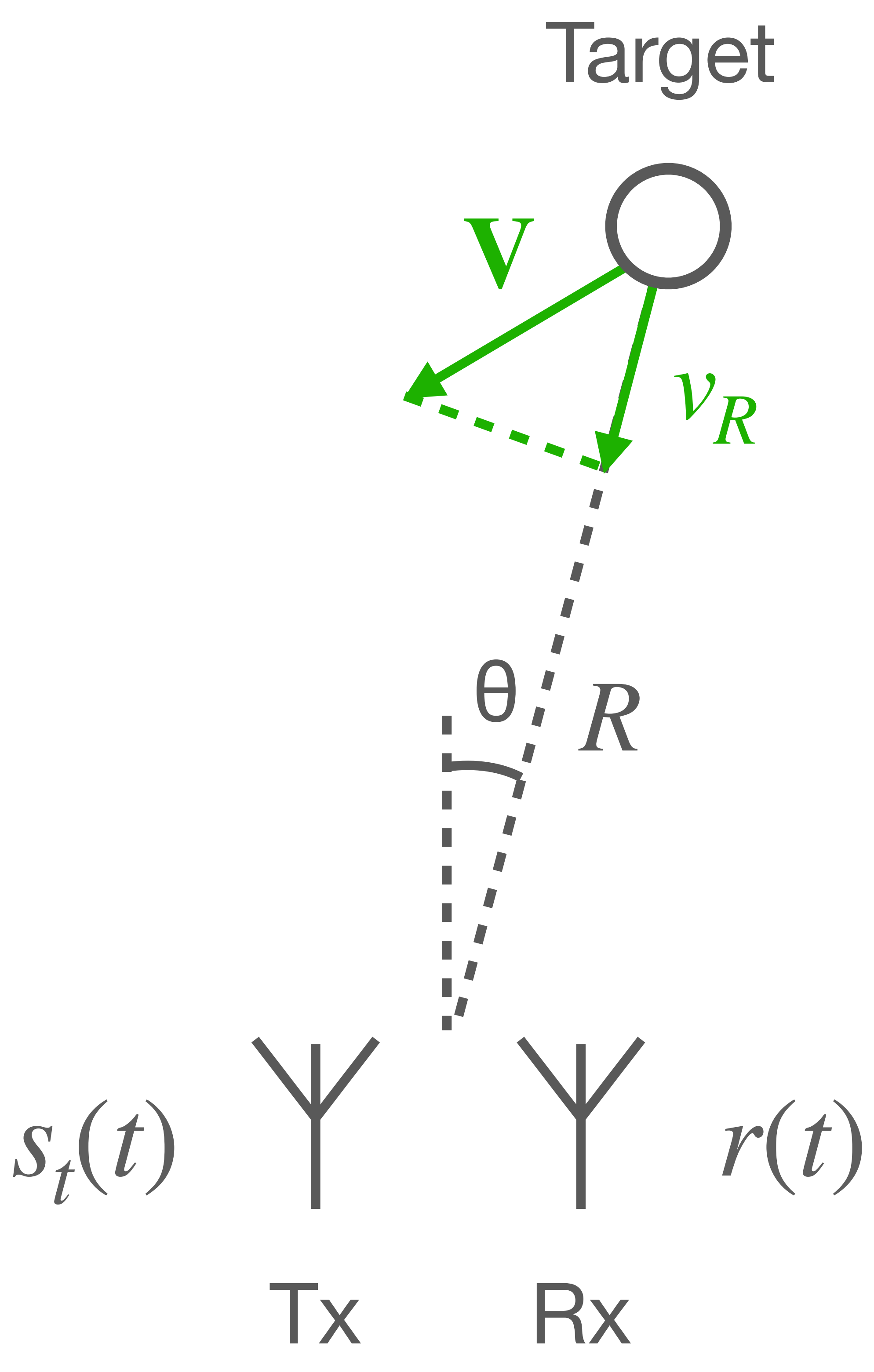
Doppler-shift found by differentiation of phase of $r_d(t)$

$$f_d(t) = \frac{1}{2\pi}\frac{d\phi_{r_d}(t)}{dt} = -\frac{d}{dt}f_0\tau_d$$

Because R is time-dependent, $\frac{d}{dt}\tau_d = \frac{2v_R}{c}$

$$f_d(t) = -\frac{2v_R}{\lambda} \implies \boxed{v_R = -f_d\frac{\lambda}{2}}$$

where λ is the wavelength



Bistatic Radar

Range-Doppler Measurement

Current Methods

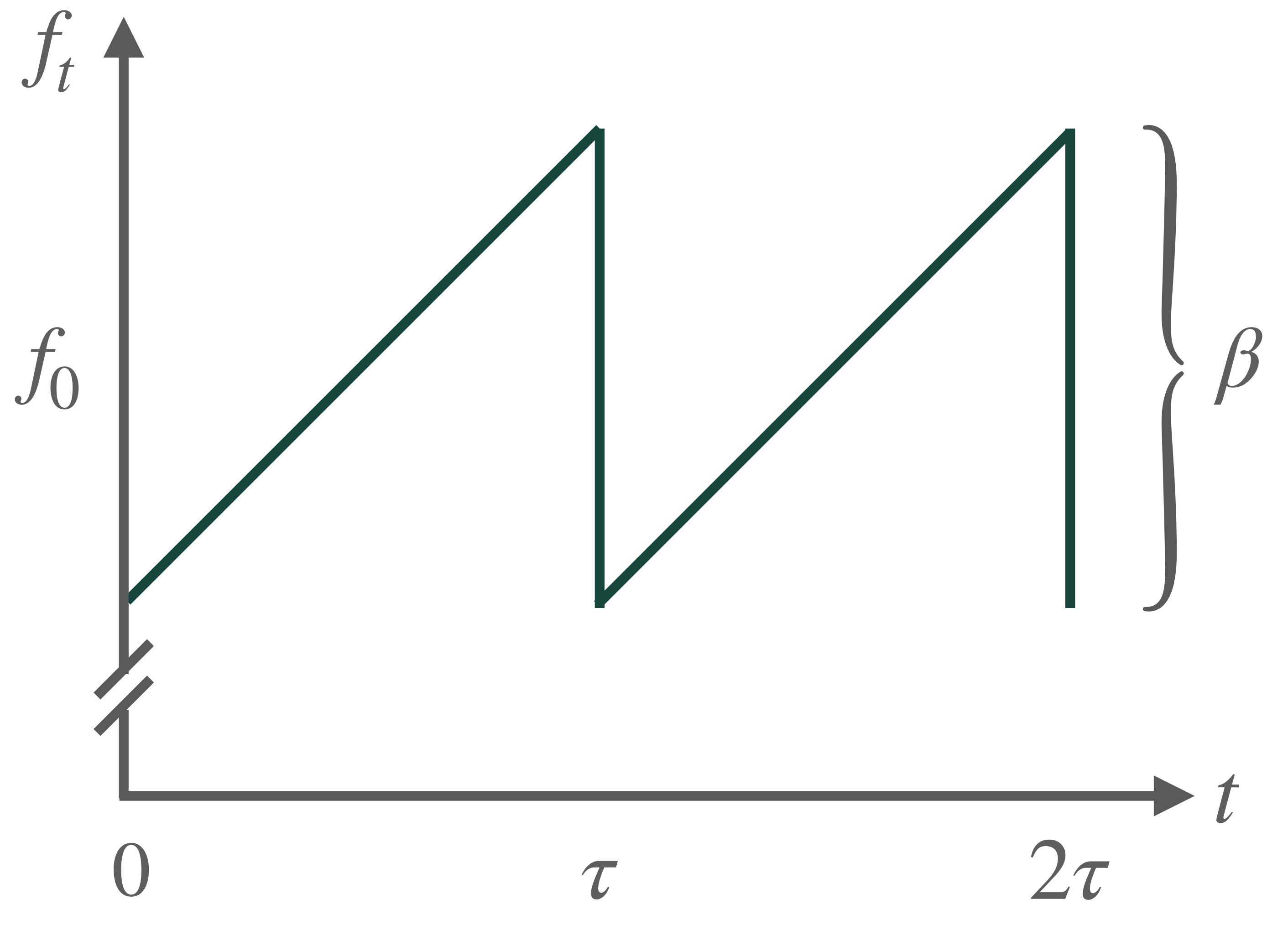
- With modulation, range *and* velocity can be obtained
- Linear frequency modulation (LFM) is commonly used due to its ease of implementation

$$\omega_t(t) = 2\pi\left(f_0 + Kt\right);\ t \in [-\tau/2, \tau/2]$$

where $K = \beta/\tau$ is the chirp rate
β is the chirp bandwidth
τ is the chirp duration

$$s_t(t) = A(\theta)\ \exp\left[\left[\int \omega_s(t)dt\right]\right] = A(\theta)\ \exp\left[j2\pi\left(f_0 t + \frac{K}{2}t^2\right)\right]$$

Transmitted Frequency Vs. Time



Range-Doppler Measurement

Current Methods

$$s_t(t) = A(\theta)\ \exp\left[j2\pi\left(f_0 t + \frac{K}{2}t^2\right)\right]$$

Received signal at r_n:

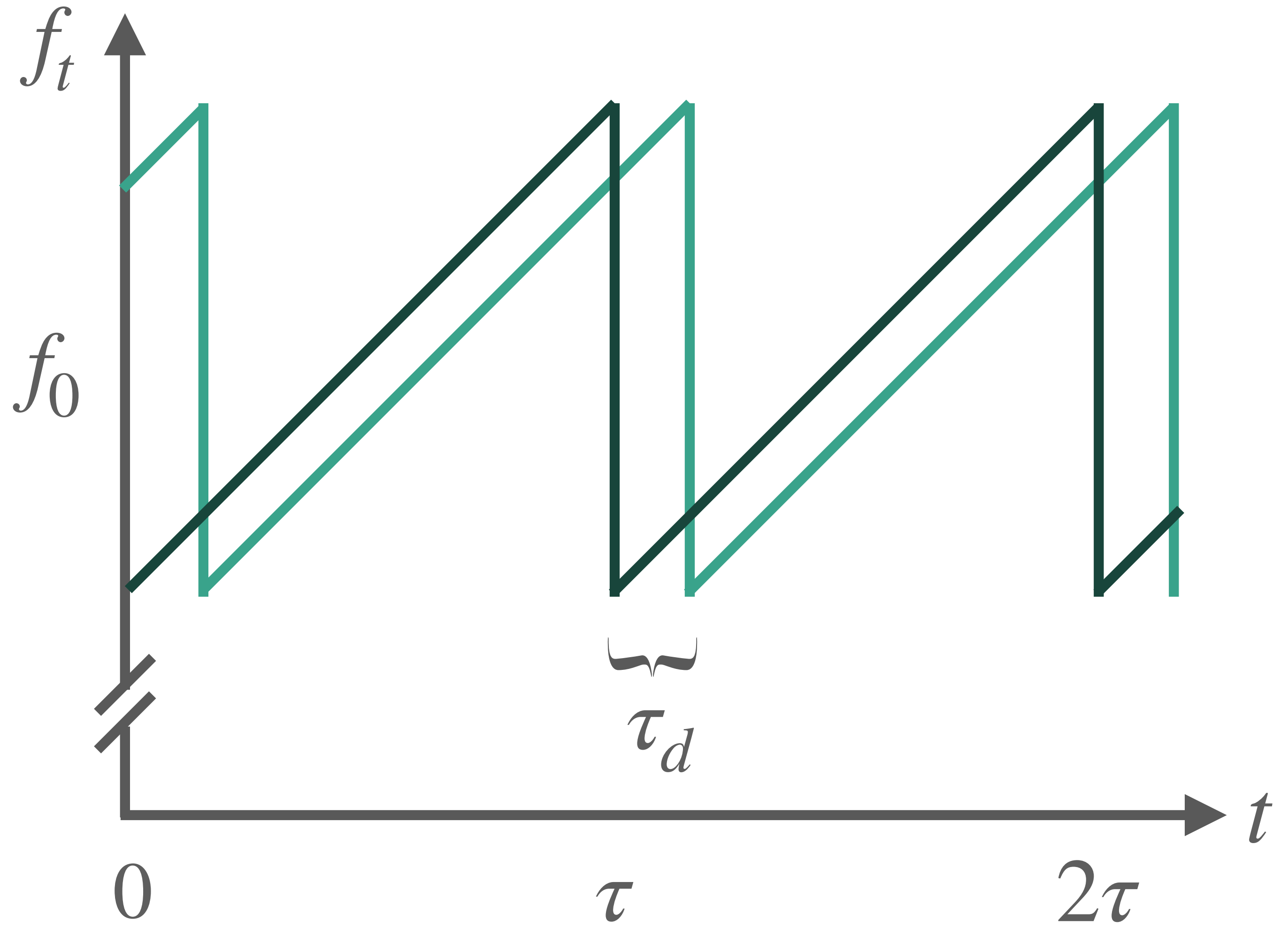
$$r_n = A(\theta)\ \exp\left\{j2\pi\left[f_0\left(t-\tau_{dn}\right) + \frac{K}{2}\left(t-\tau_{dn}\right)^2\right]\right\}$$

where $\tau_{dn} = \dfrac{2R_n}{c}$

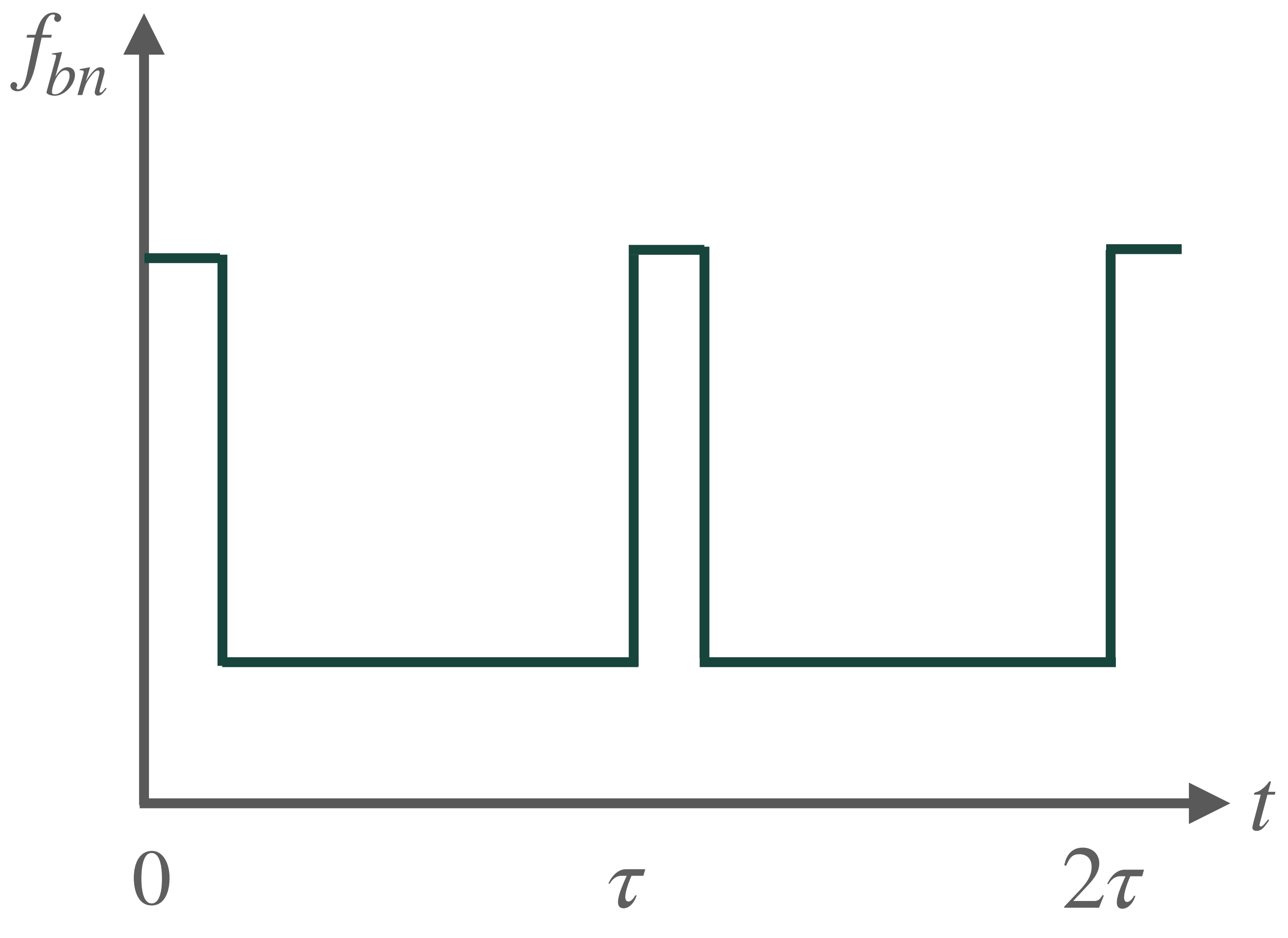
Downconverted signal at r_{bn}:

$$\begin{aligned} r_{bn}(t) &= r_n(t)\cdot s_t^*(t) \\ &= A\left(\theta\right)\exp\left\{-j2\pi\left[f_0\tau_{dn} + \frac{K}{2}\left(\tau_{dn}^2 - 2\tau_{dn}t\right)\right]\right\} \end{aligned}$$

Transmitted & Received Frequencies Vs. Time



Beat Frequency Vs. Time



Range-Doppler Measurement

Current Methods

$$r_{bn}(t) = A(\theta)\exp\left\{-j2\pi\left[f_0\tau_{dn} + \frac{K}{2}\left(\tau_{dn}^2 - 2\tau_{dn}t\right)\right]\right\}$$

Beat frequency found by differentiation of phase of r_{bn}

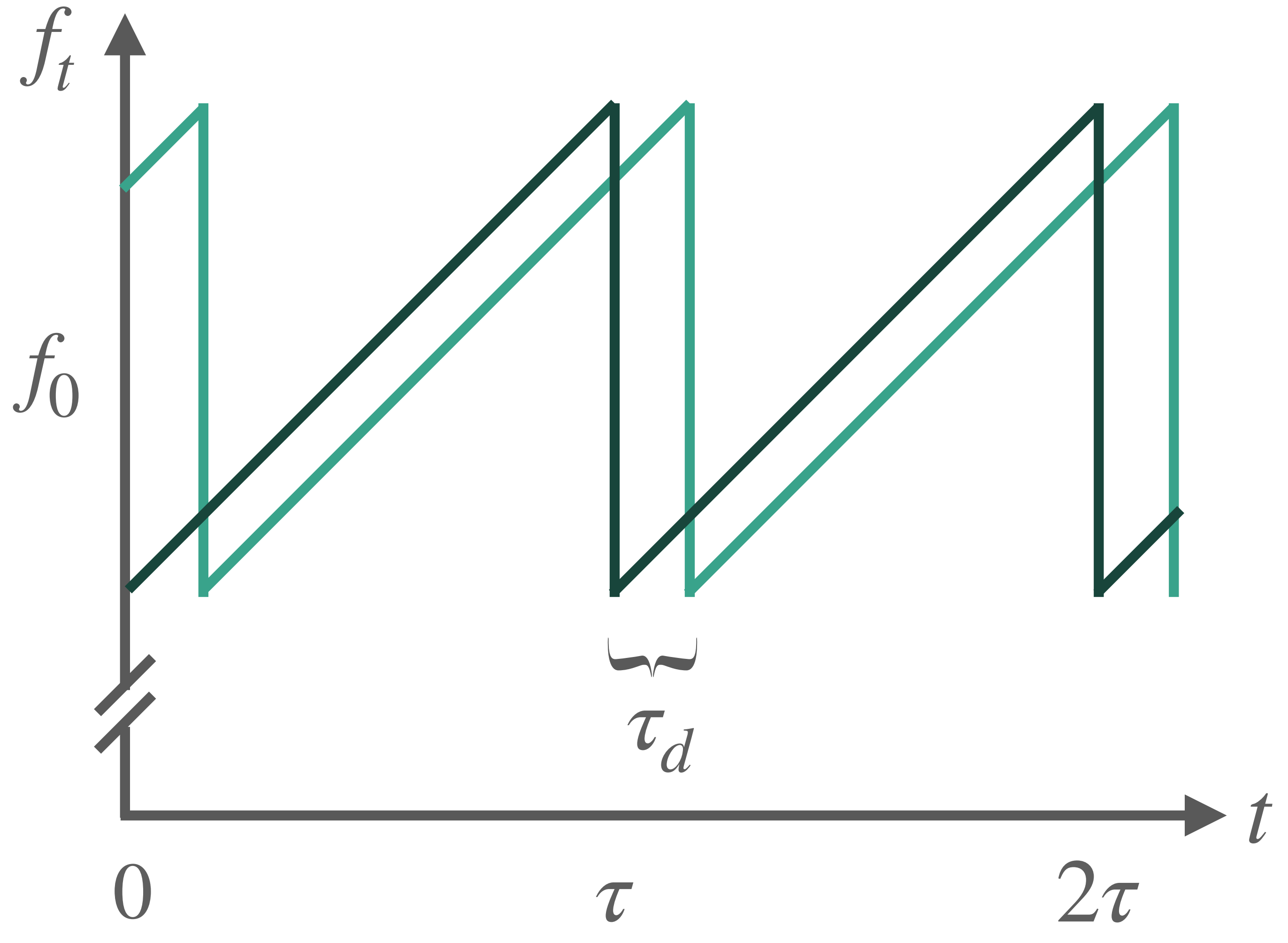
≪ range term

$$f_{bn} = \frac{1}{2\pi}\frac{d\phi_{r_{bn}}(t)}{dt} = -\frac{2v_R}{\lambda} - \frac{2}{c}K\left(R + v_R t - \frac{4}{c^2}Rv_R\right)$$

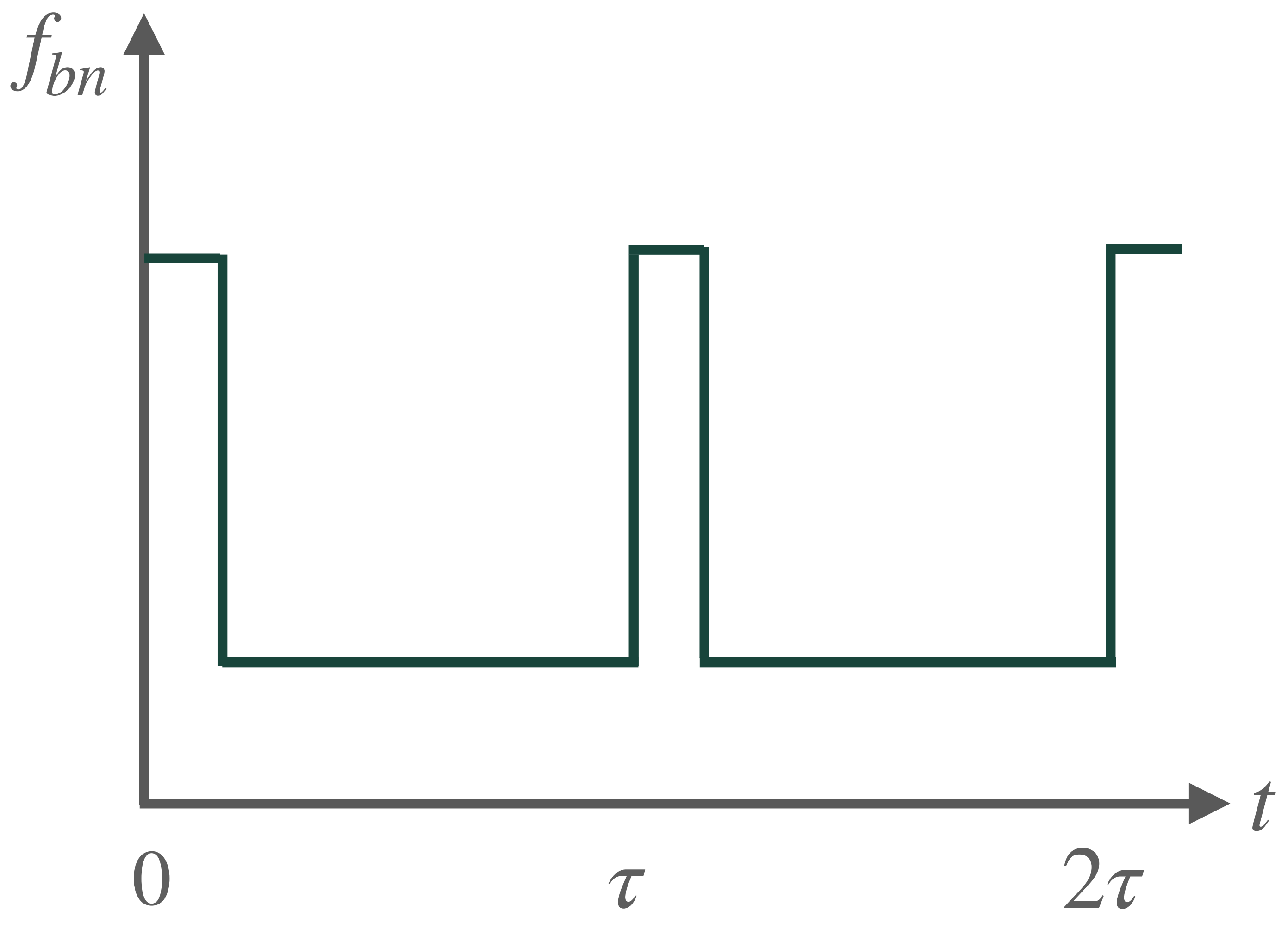
Doppler Velocity | **Range** | **Intermodulation Terms**

$$\implies \boxed{R = -f_{bn}\frac{c}{2K}} \quad \text{for } v_r = 0$$

Transmitted & Received Frequencies Vs. Time



Beat Frequency Vs. Time



Range-Doppler Measurement

Current Methods

$$r_{bn}(t) = A(\theta) \exp \left\{ -j2\pi \left[f_0 \tau_{dn} + \frac{K}{2} \left(\tau_{dn}^2 - 2\tau_{dn} t \right) \right] \right\}$$

Beat frequency found by differentiation of phase of r_{bn}

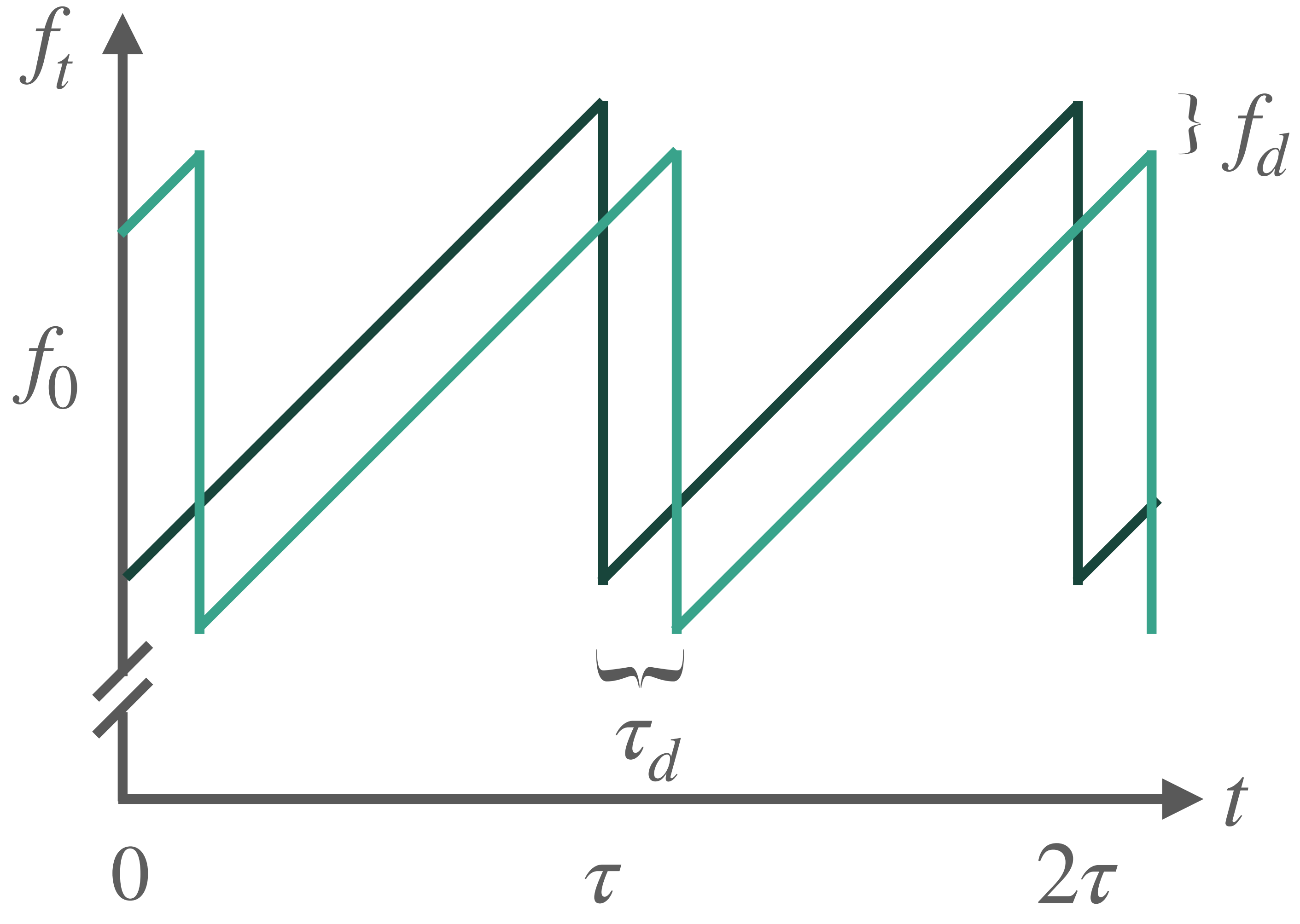
≪ range term

$$f_{bn} = \frac{1}{2\pi} \frac{d\phi_{r_{bn}}(t)}{dt} = -\underbrace{\frac{2v_R}{\lambda}}_{\text{Doppler Velocity}} - \underbrace{\frac{2}{c}K\Big(R}_{\text{Range}} + \underbrace{v_R t - \frac{4}{c^2} R v_R}_{\text{Intermodulation Terms}}\Big)$$

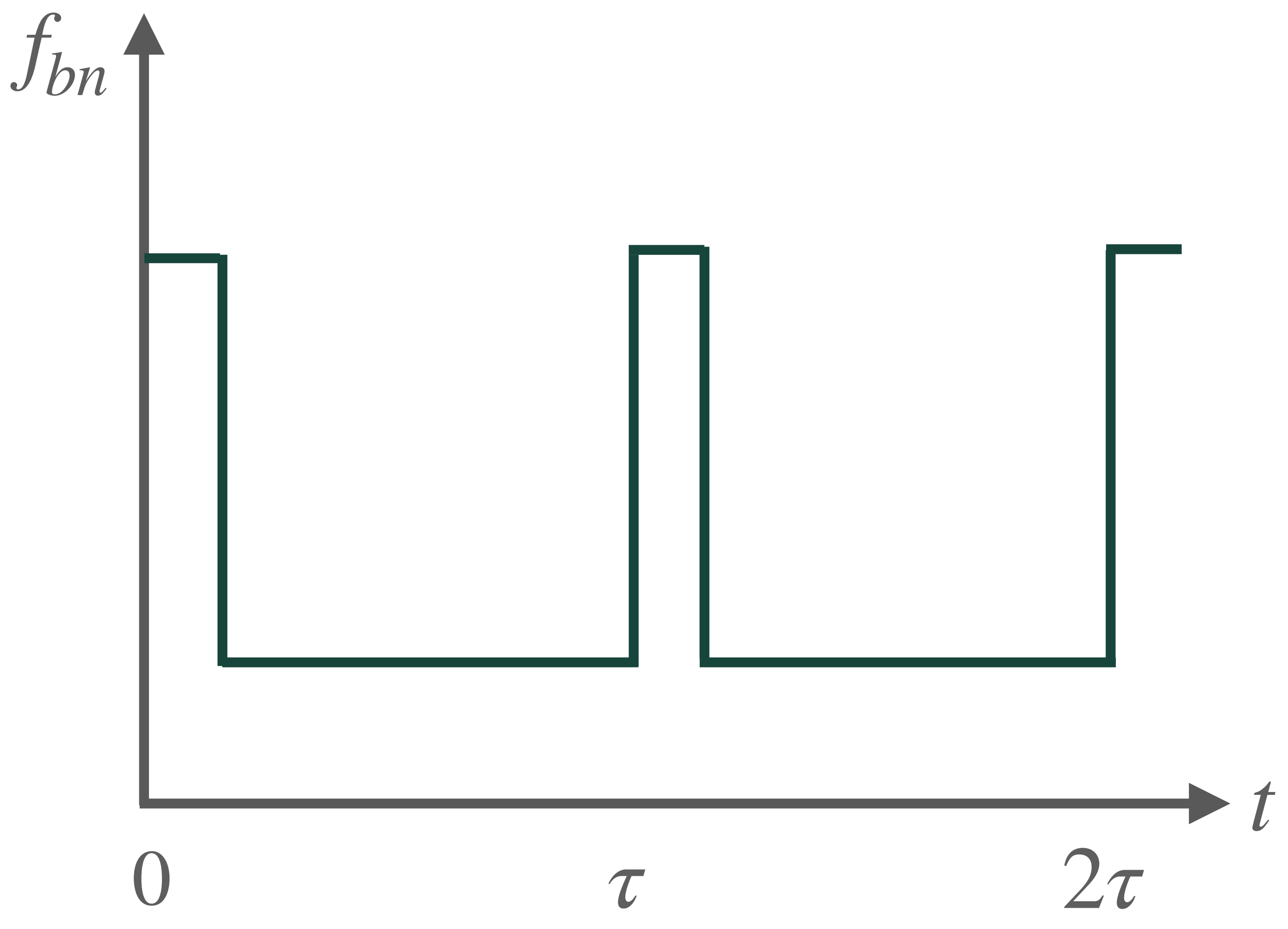
Doppler Velocity **Range** **Intermodulation Terms**

$$\implies \boxed{R \approx -f_{bn} \frac{c}{2K}}$$ under quasi-static assumption

Transmitted & Received Frequencies Vs. Time



Beat Frequency Vs. Time



Range-Doppler Measurement

Current Methods

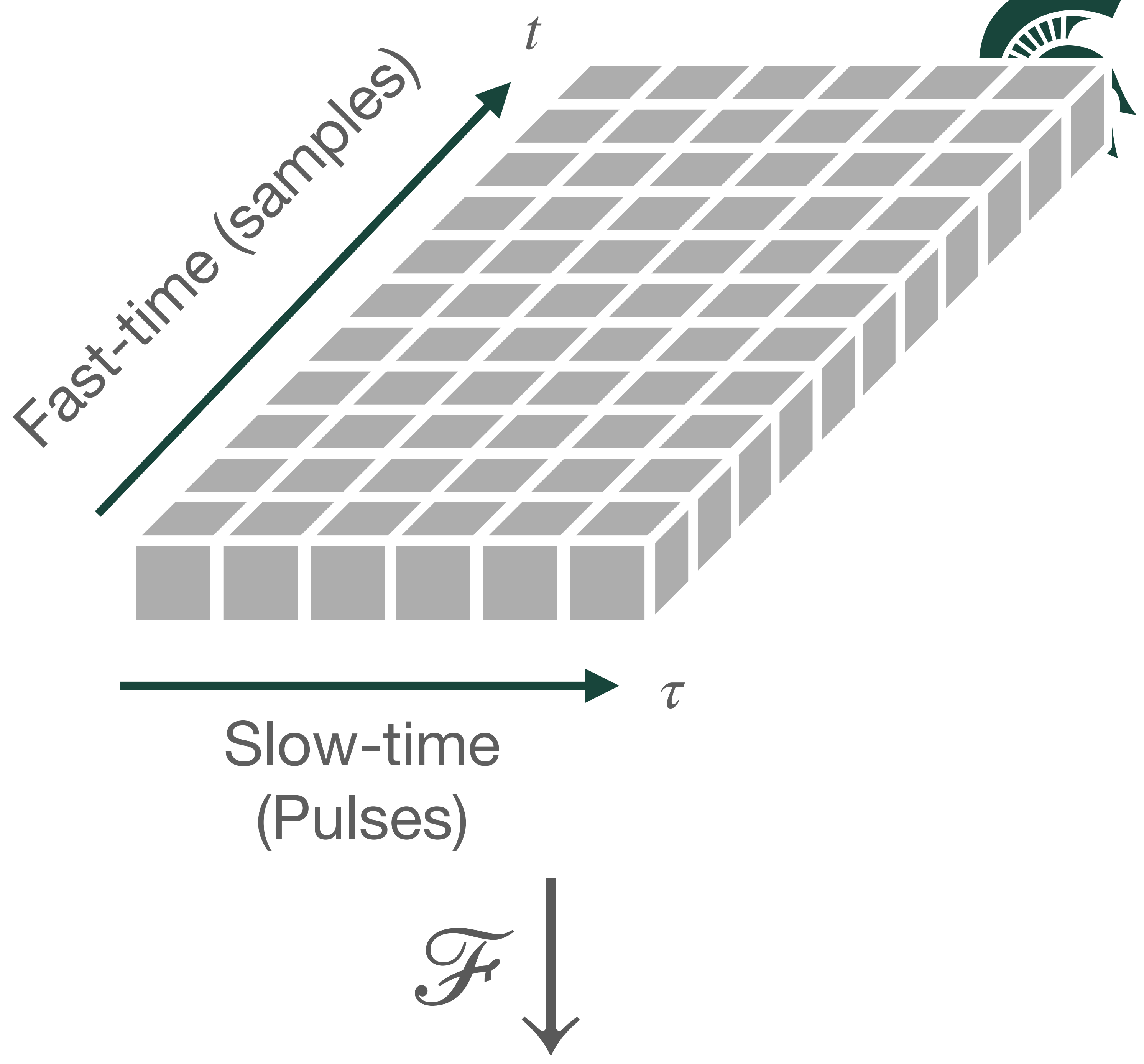
Doppler shift from moving LFM scatterer:

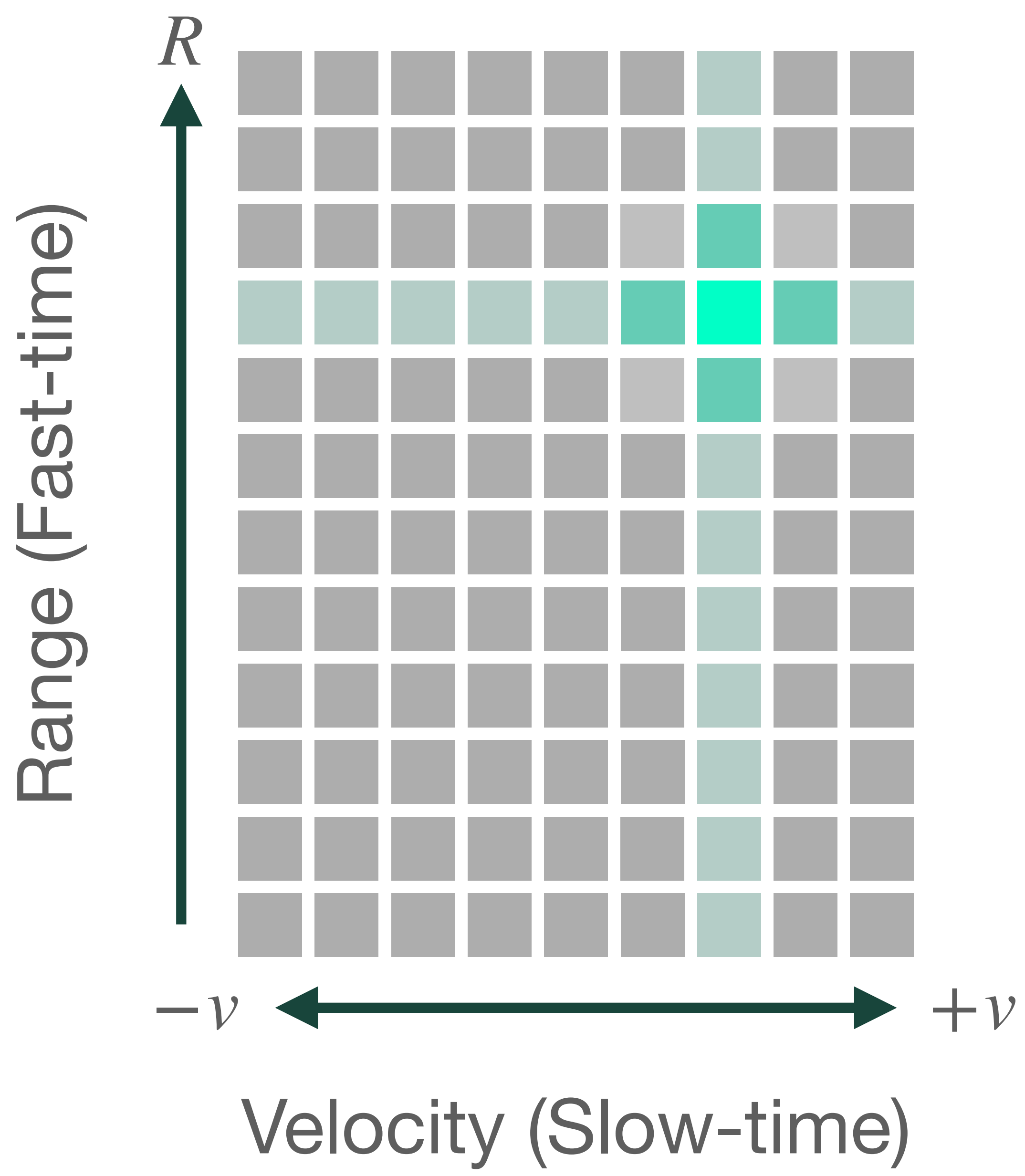
$$f_d = \frac{1}{2\pi}\Delta_{bn}(t) = f_0\Delta\tau_{dn} + \frac{K}{2}\left[\tau_{dn_1}^2 - \tau_{dn_2}^2 - 2\Delta\tau_{dn}t\right]$$

where $\Delta\tau_{dn} = \frac{2}{c}(R_2 - R_1) = -\frac{2}{c}v_{Rn}\tau$ and $v_{Rn} = -\frac{R_2 - R_1}{\tau}$

$$= -\frac{2v_R}{\lambda}\tau \quad \Longrightarrow \quad \boxed{v_R = -\frac{f_d\lambda}{2\tau}}$$

- If the PRF $\geq$ Nyquist frequency of the Doppler shift, the velocity can be resolved in the slow-time

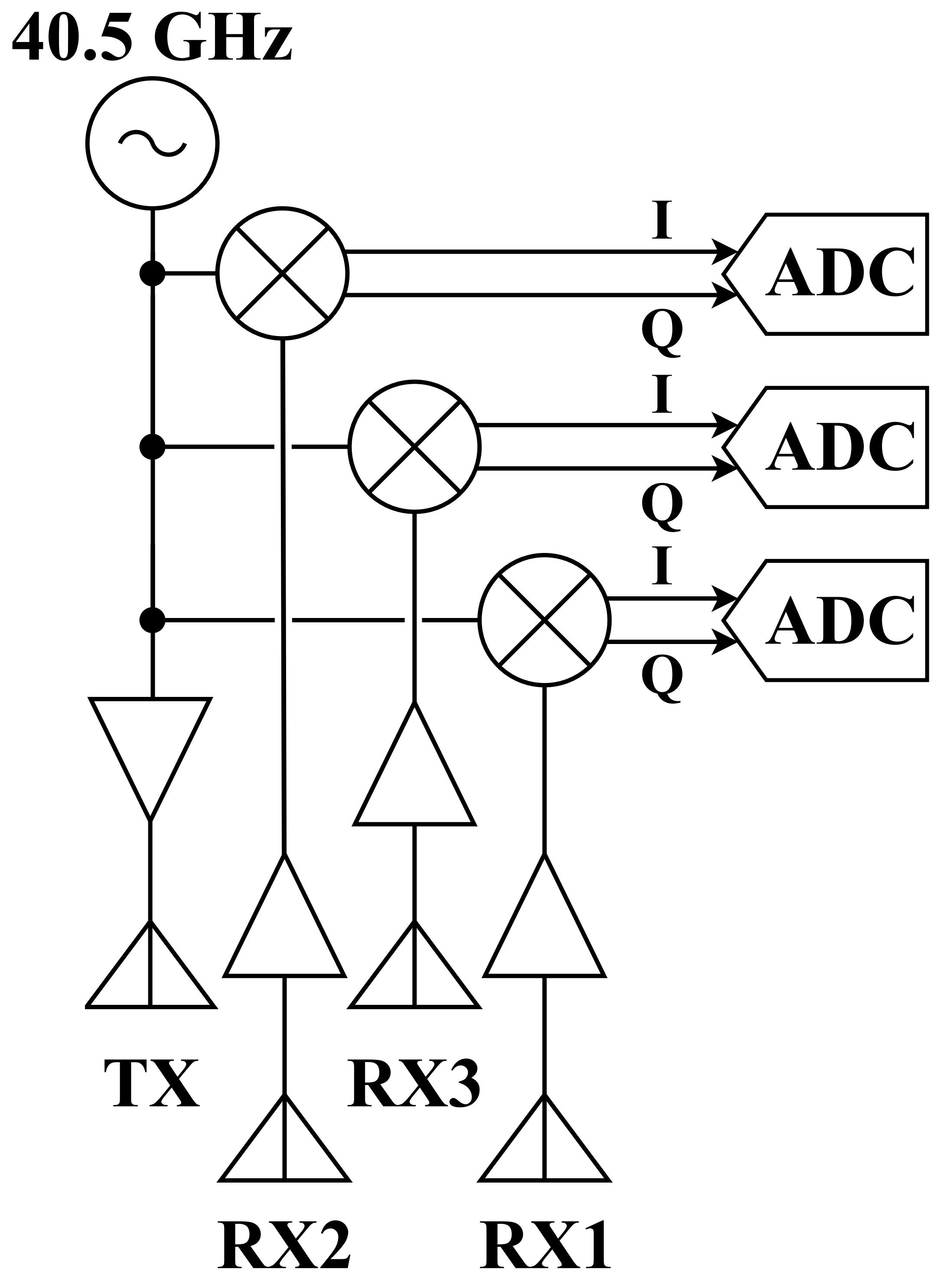


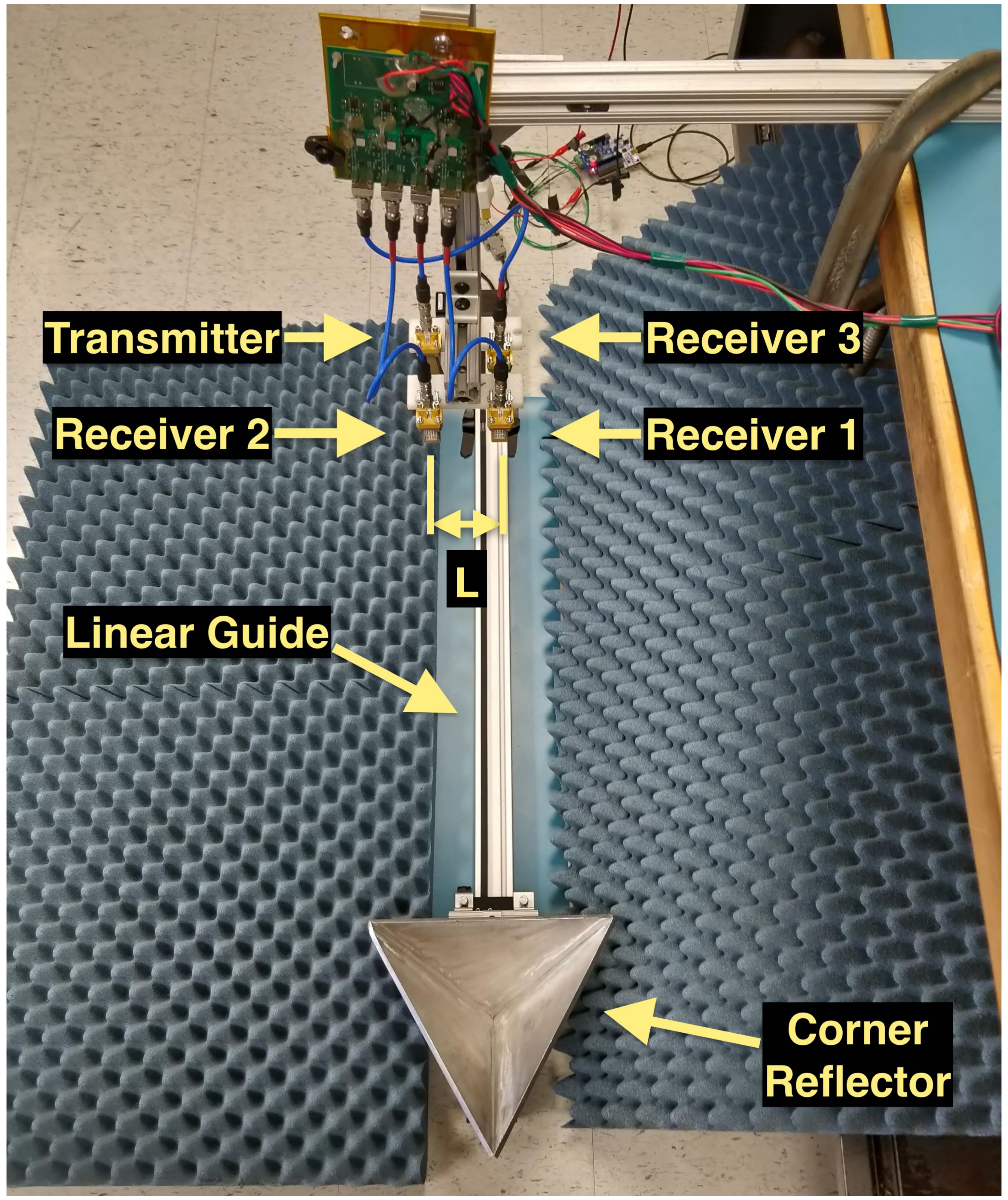


Measurement System — Radar Hardware

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

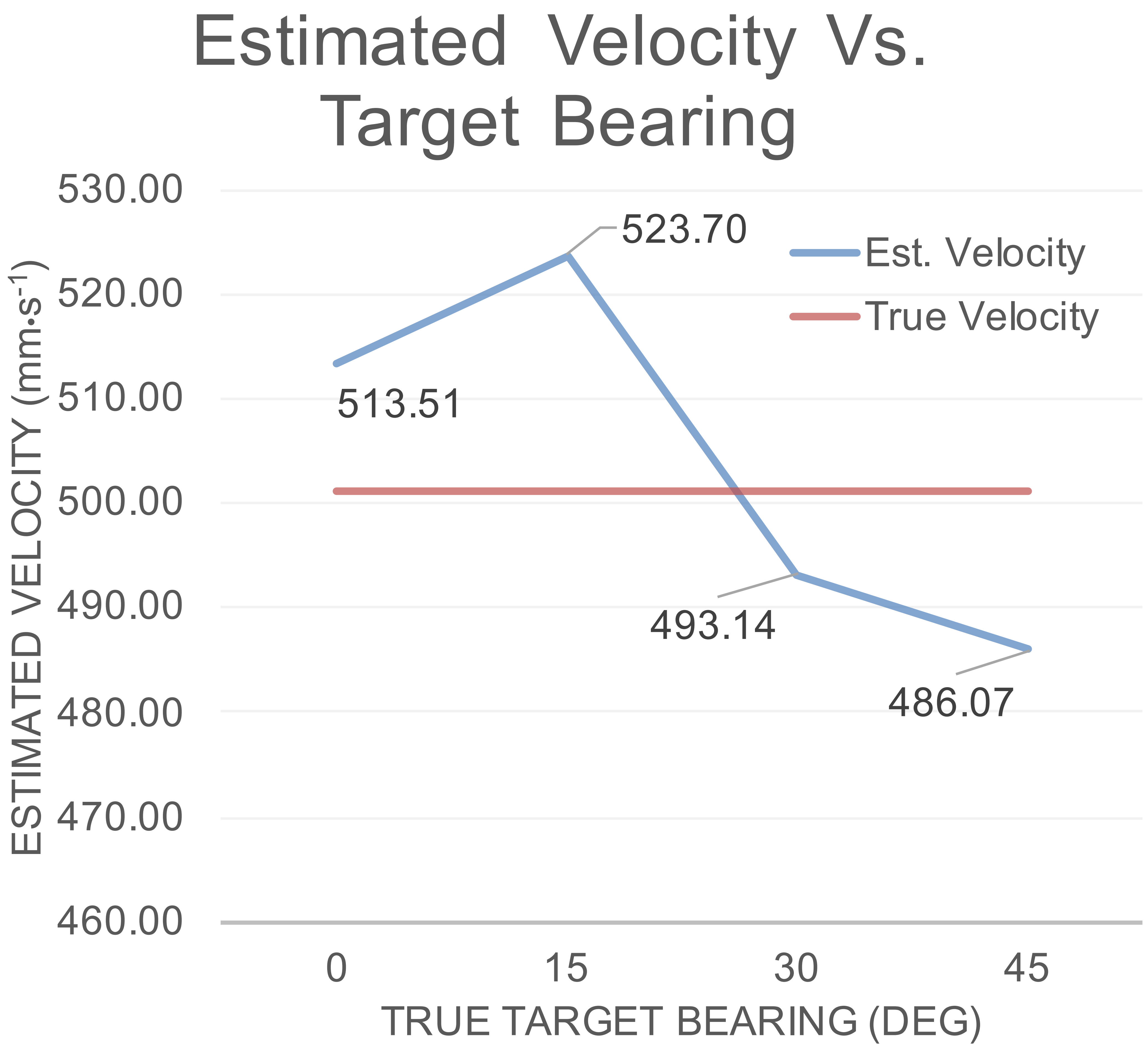
- Transmitter:
  - 40.5 GHz continuous-wave
- Antennas:
  - TX: 15 dBi
  - RX: 10 dBi
  - L = 7λ
- ADC:
  - National Instruments USB-6002 DAQ
  - Sample Rate (f_s): 4.166 kHz
- Two experimental configurations:
  - Varying bearing angle
  - Varying elevation angle

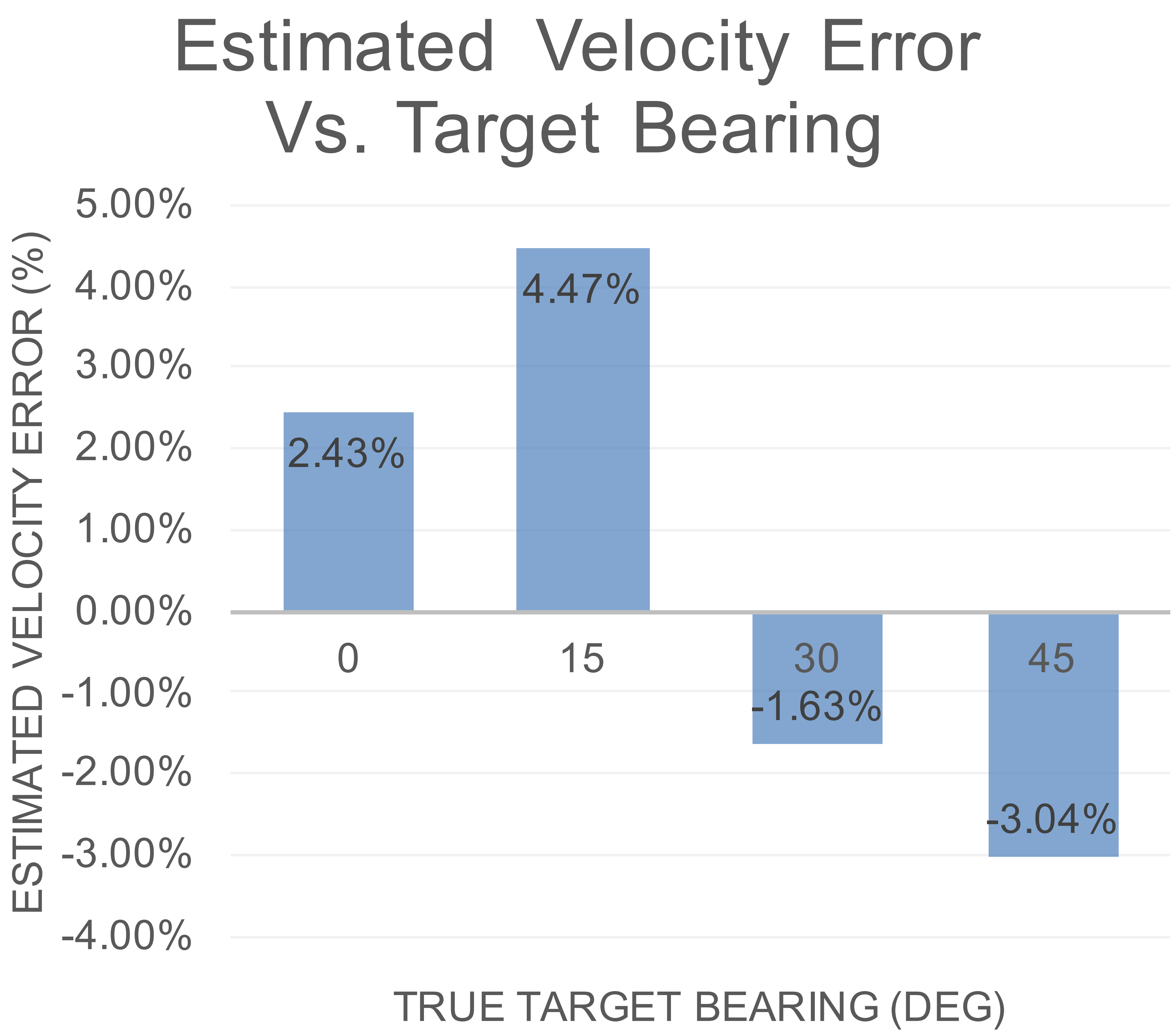




Varying Bearing - Velocity Estimates

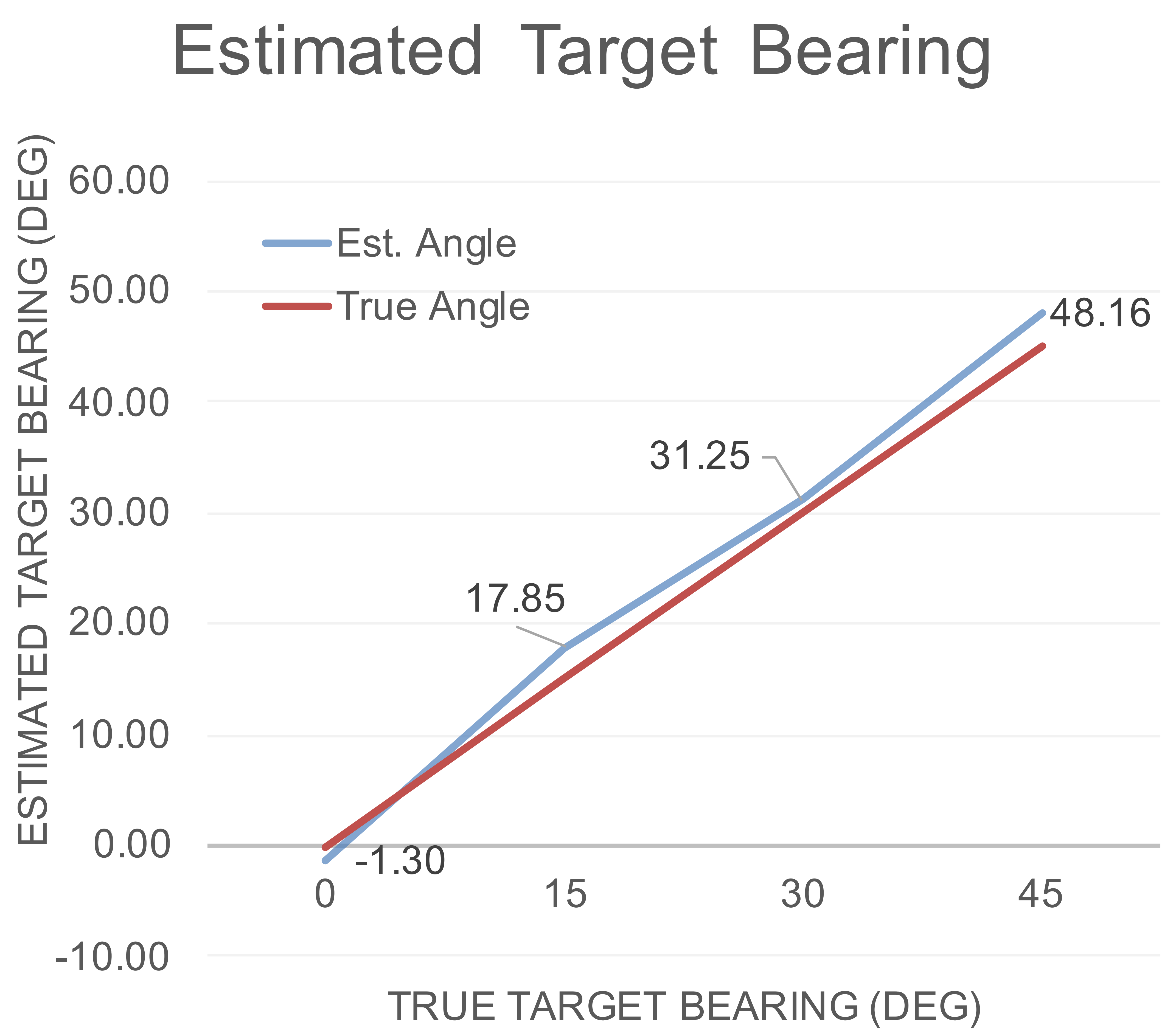
Experimental Validation - Dual-Axis Continuous-Wave Interferometer

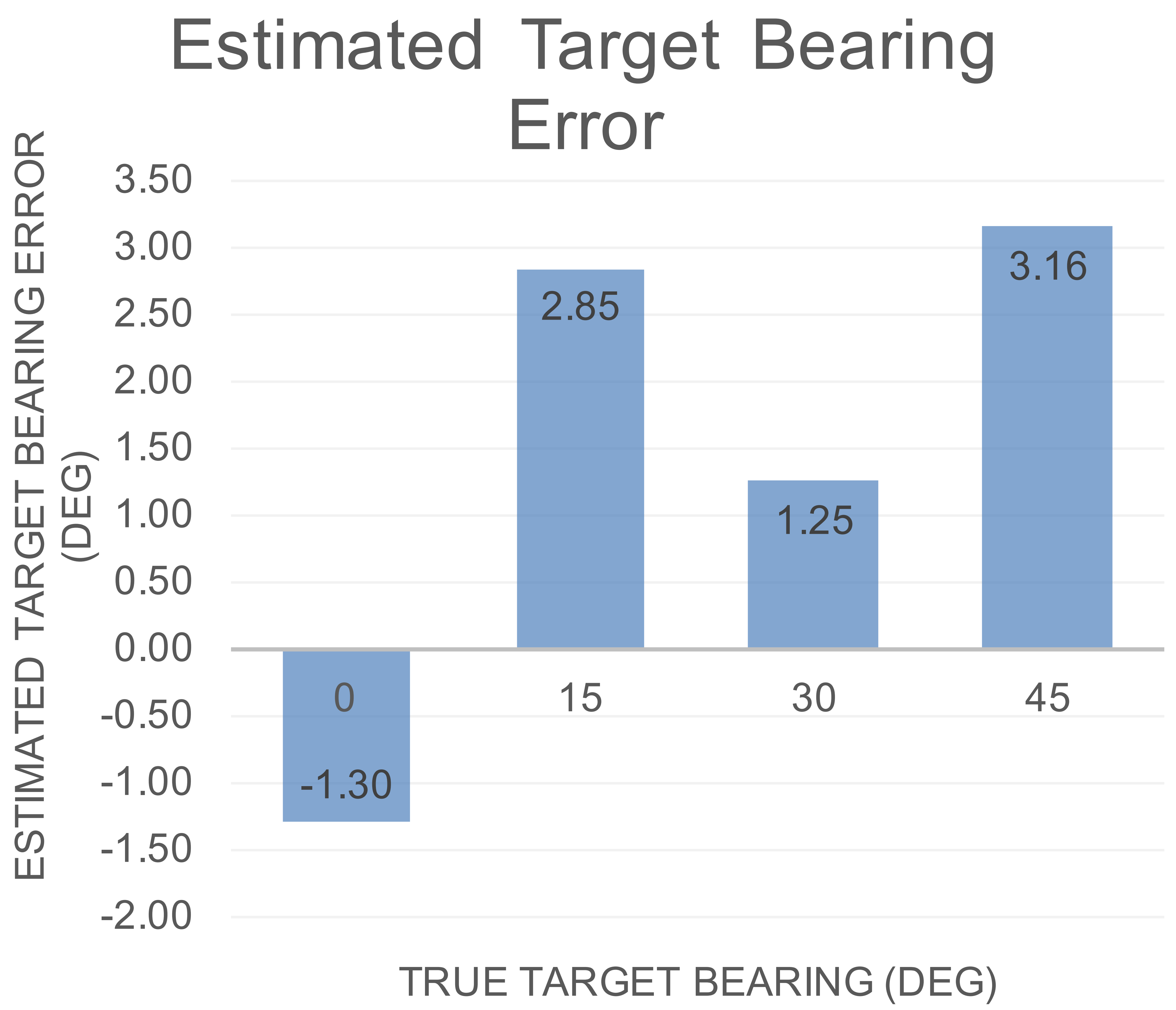




Varying Bearing - Bearing Estimates

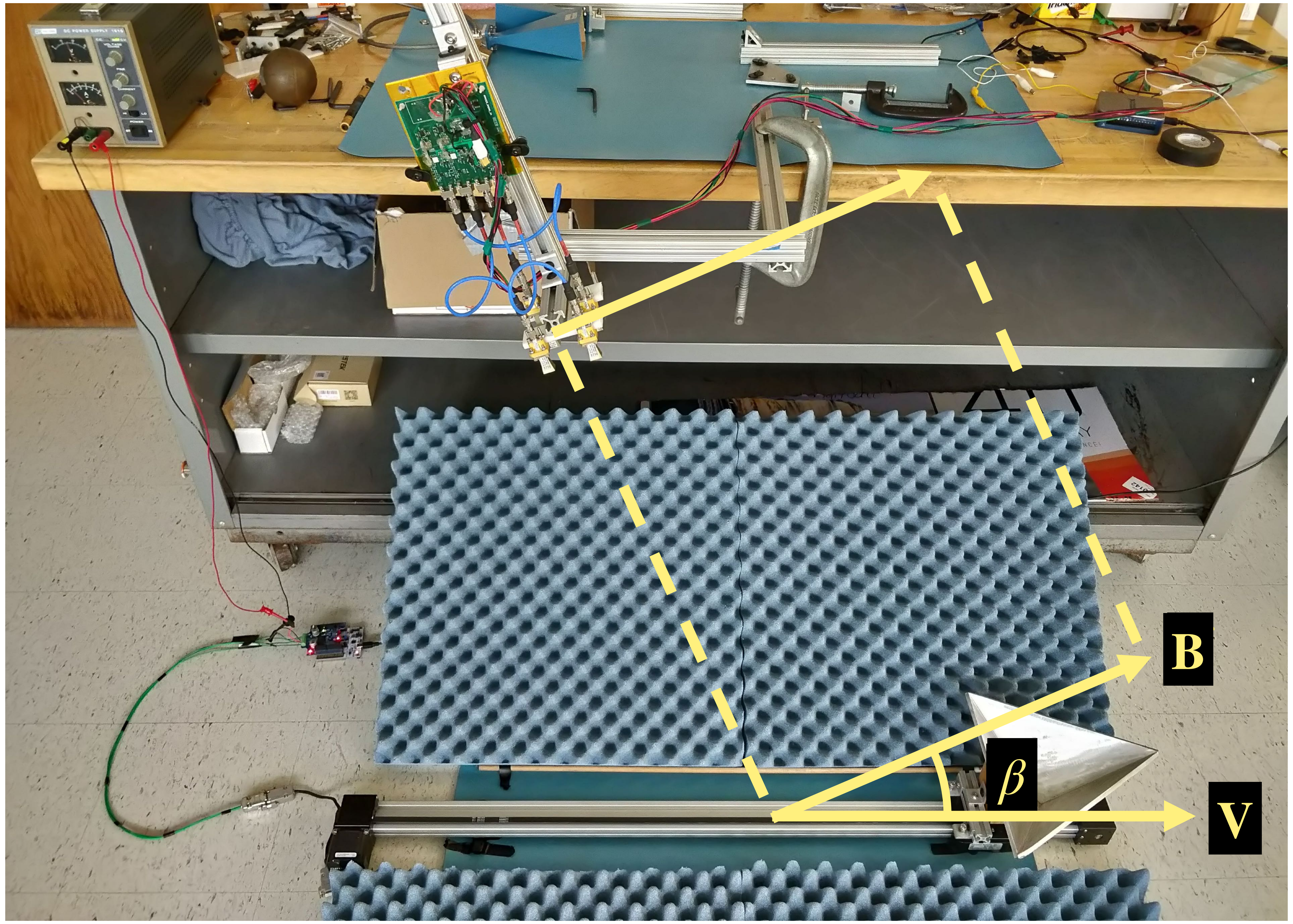
Experimental Validation - Dual-Axis Continuous-Wave Interferometer





Elevation Estimate Configuration

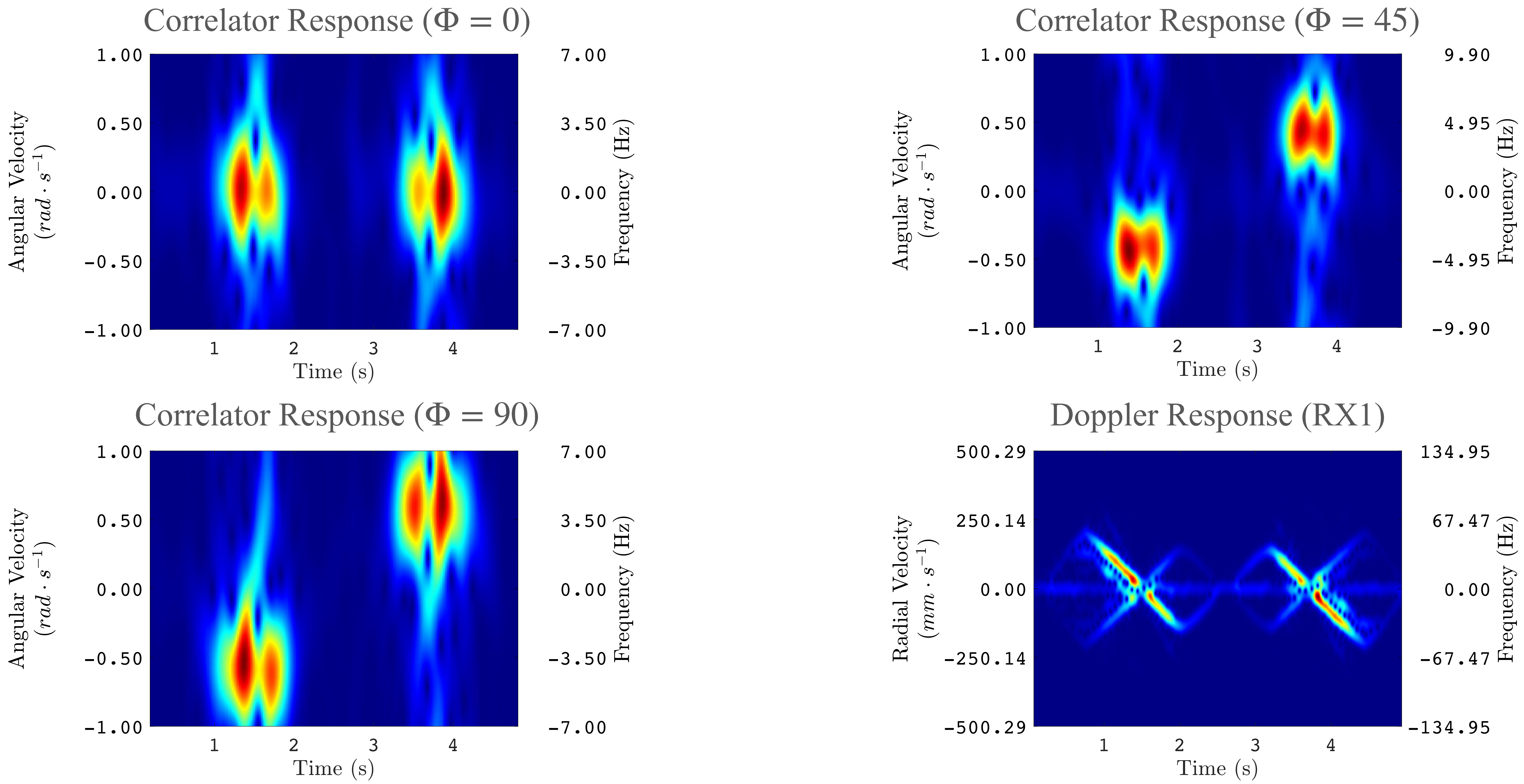
Experimental Validation - Dual-Axis Continuous-Wave Interferometer



Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

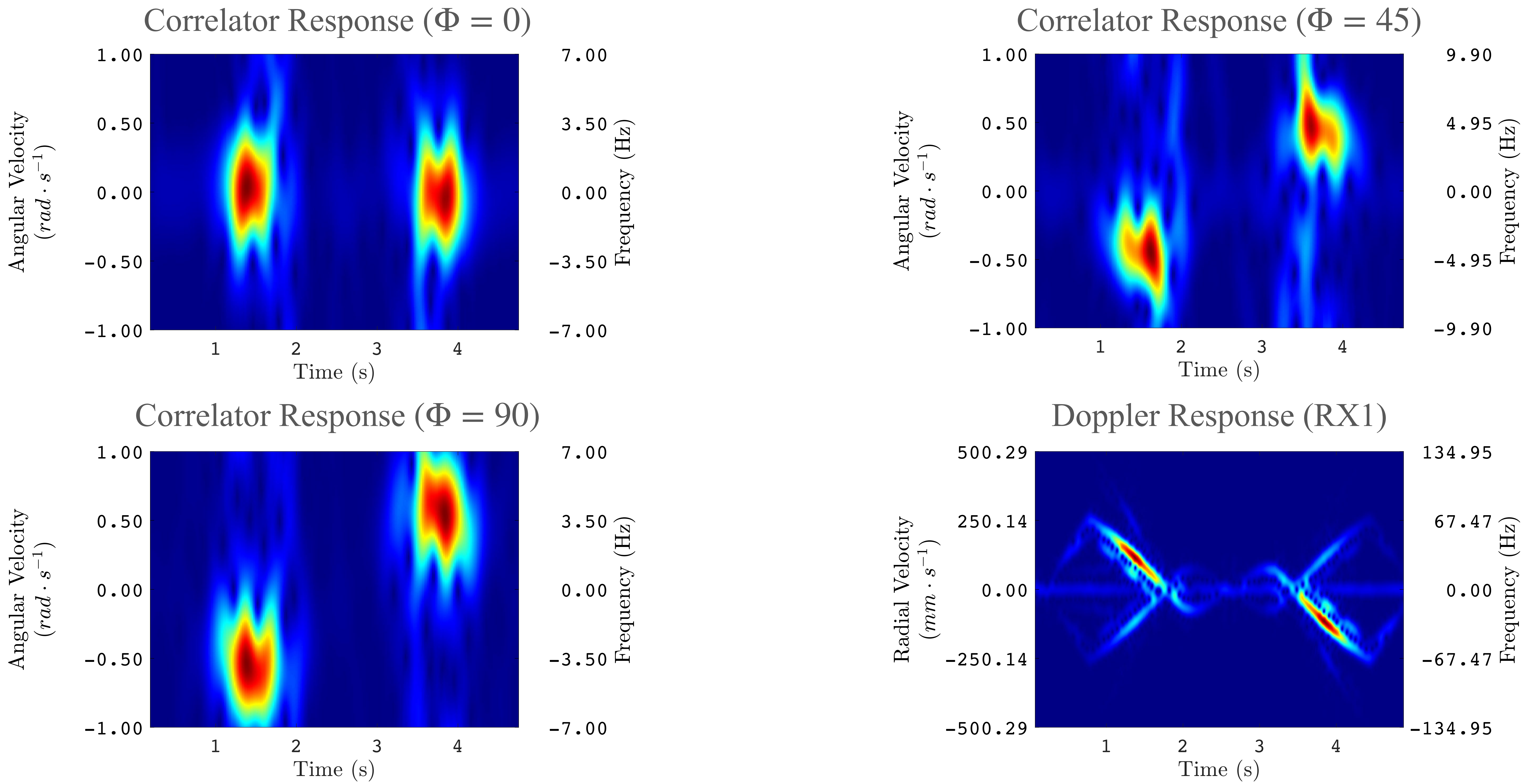
Varying β: $\beta = 0°$; $\phi = 0°$



Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

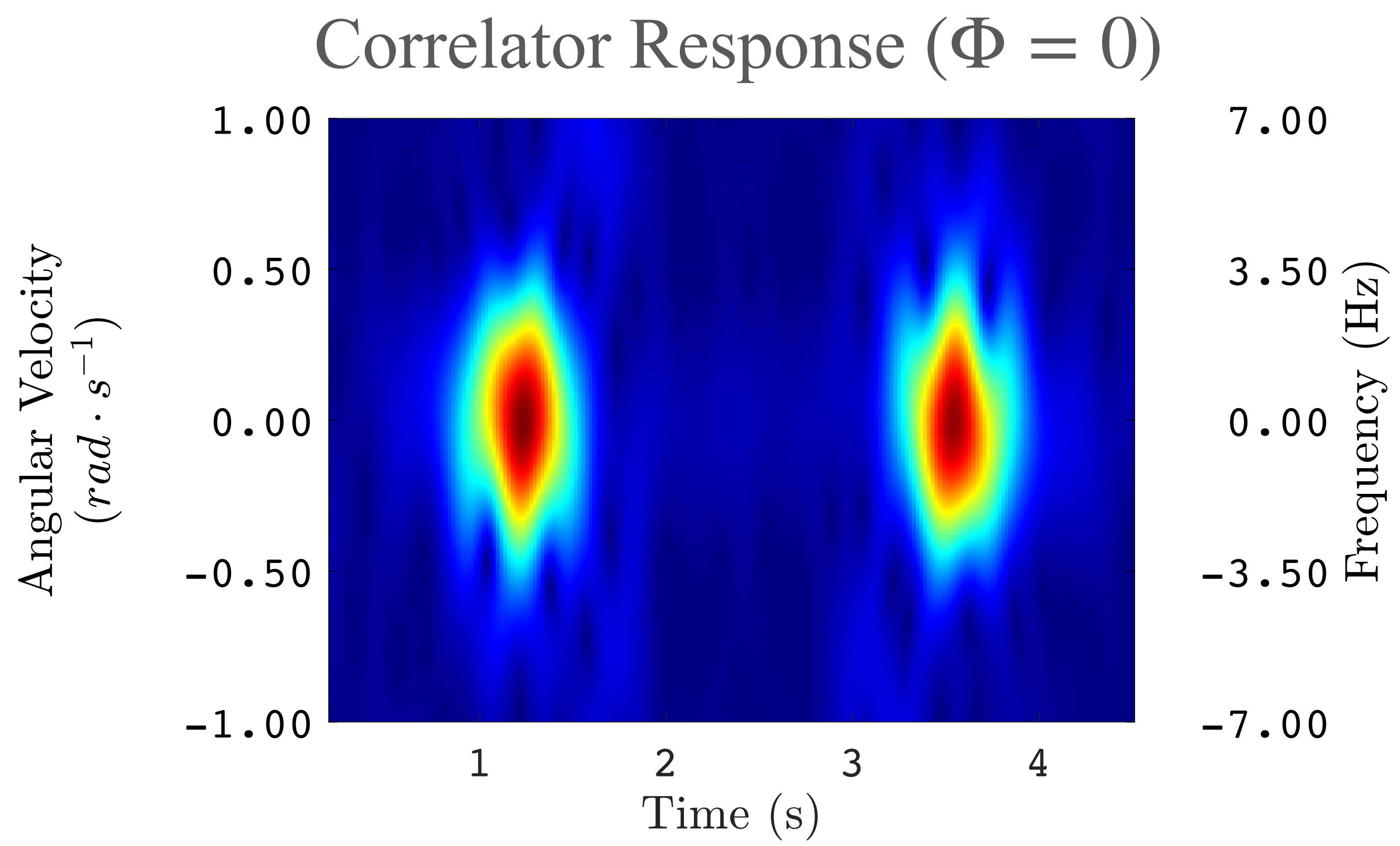
Varying β: $\beta = 10°$; $\phi = 0°$

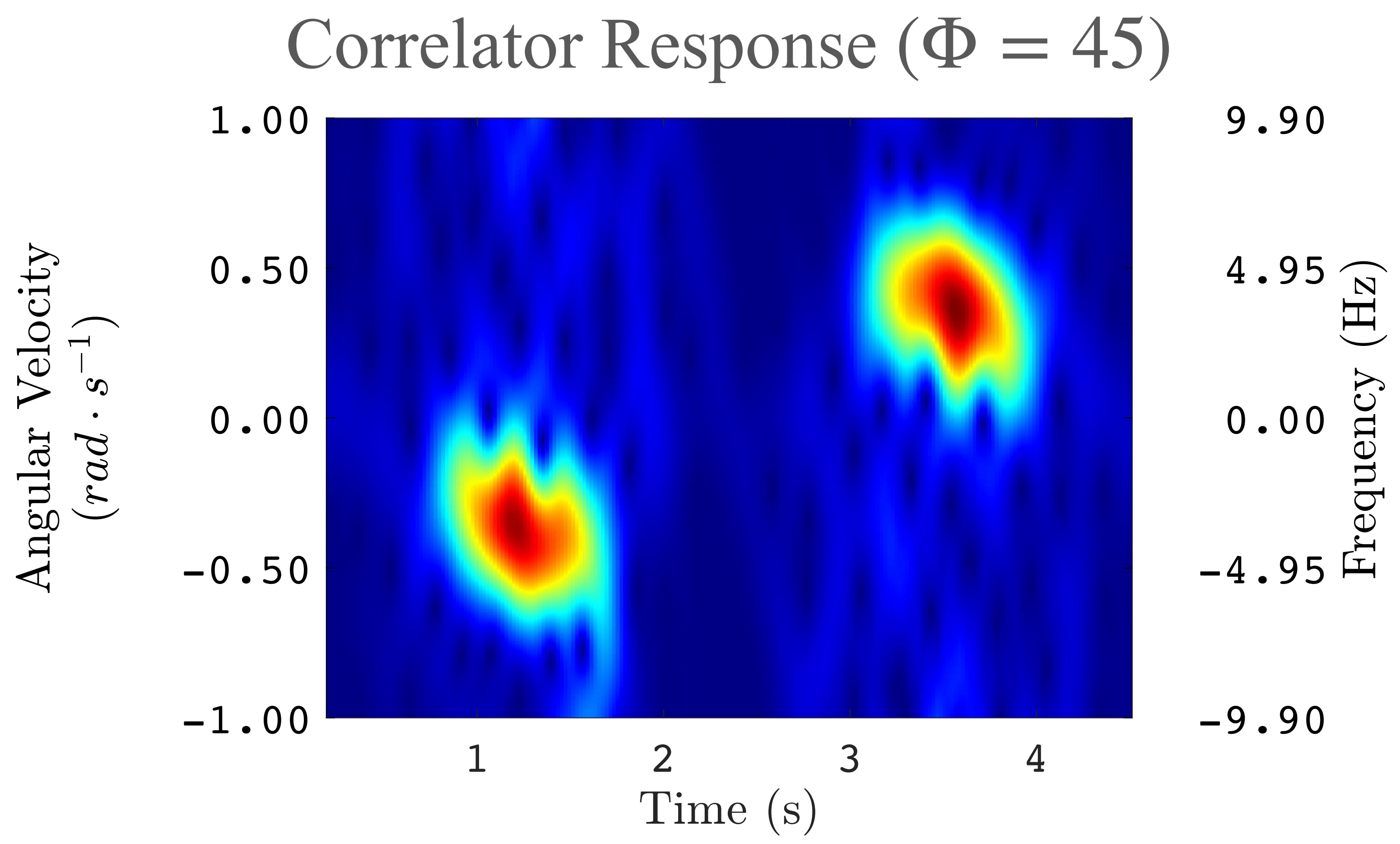


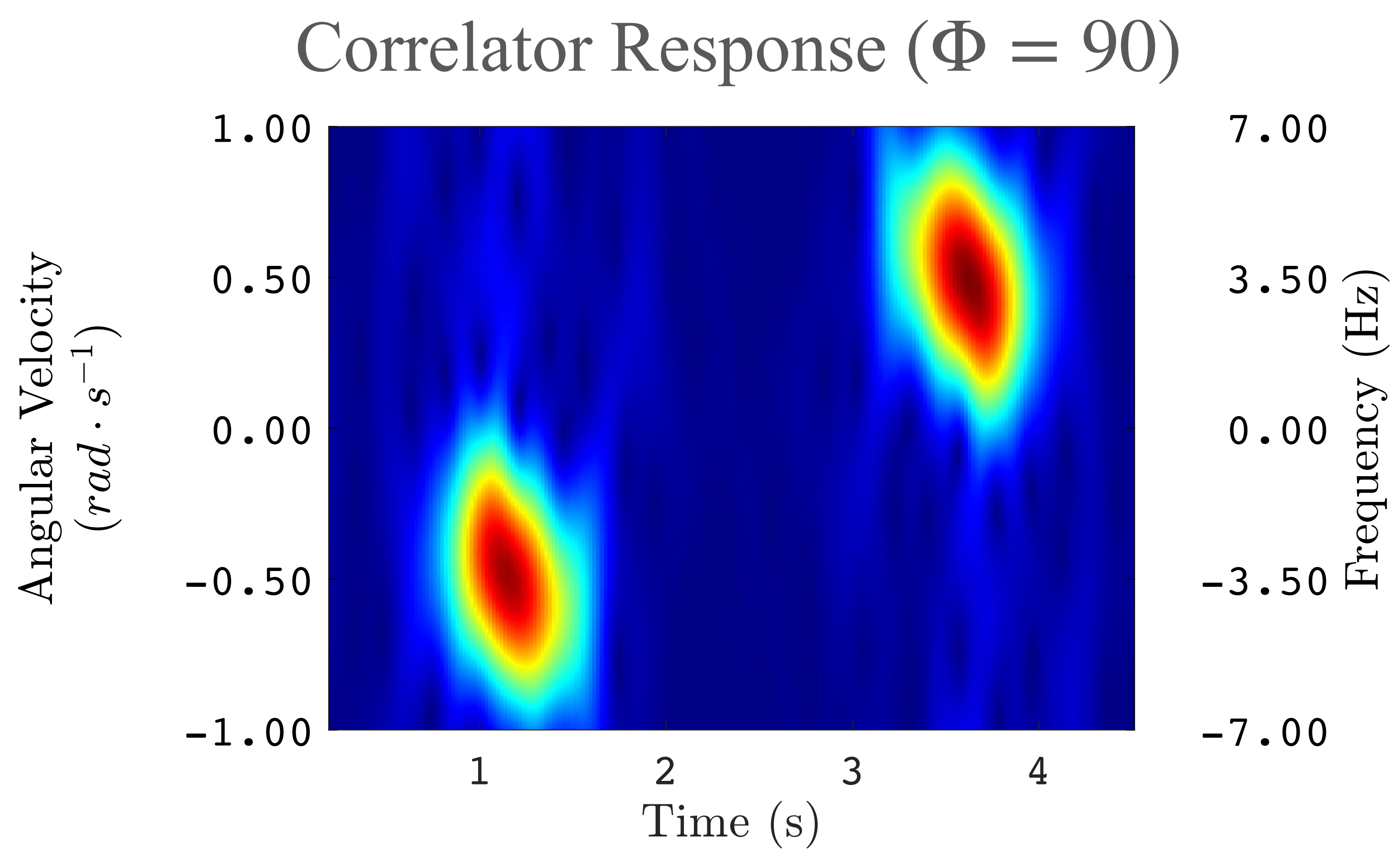
Elevation Estimate Configuration

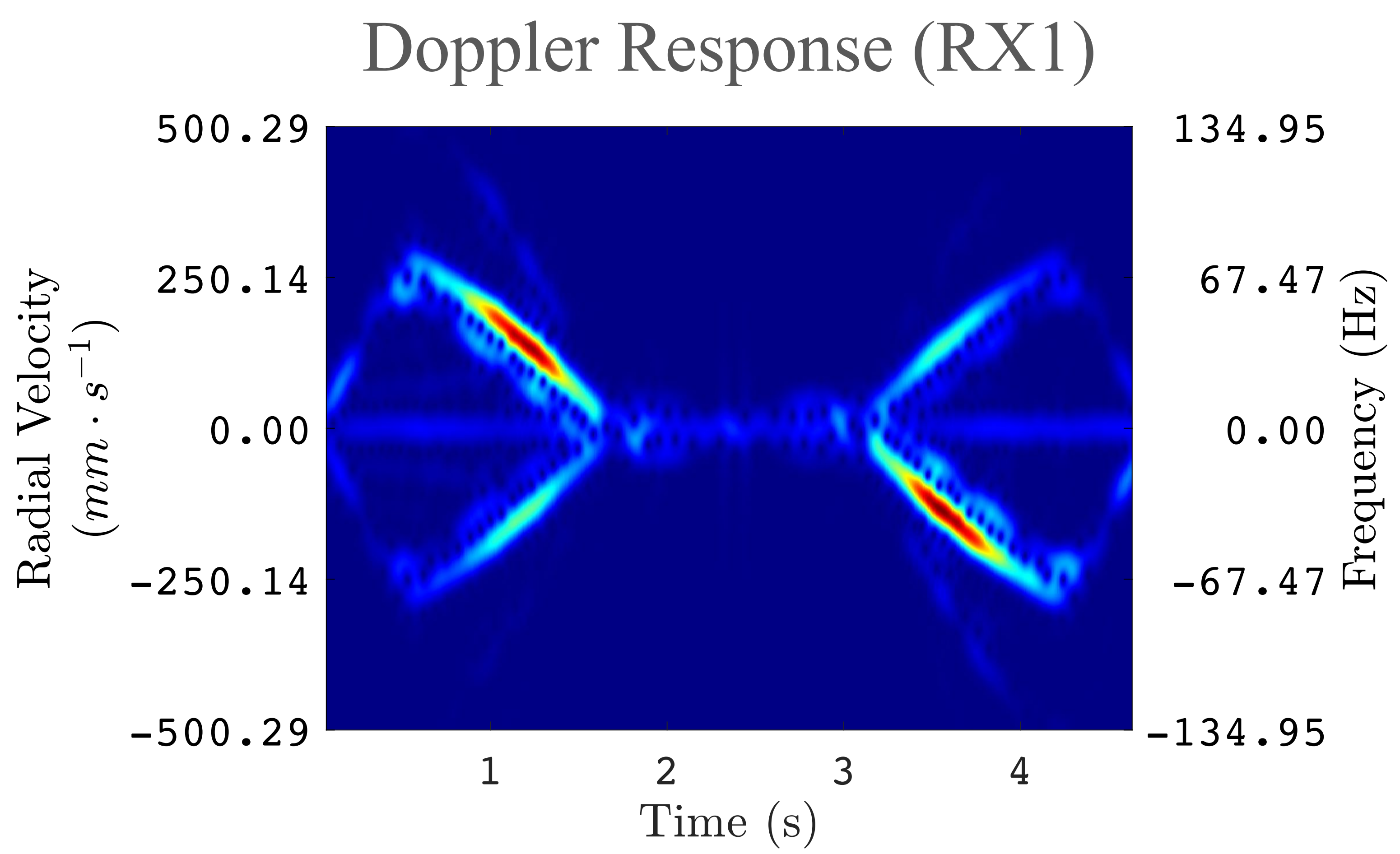
Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Varying β: $\beta = 20°$; $\phi = 0°$





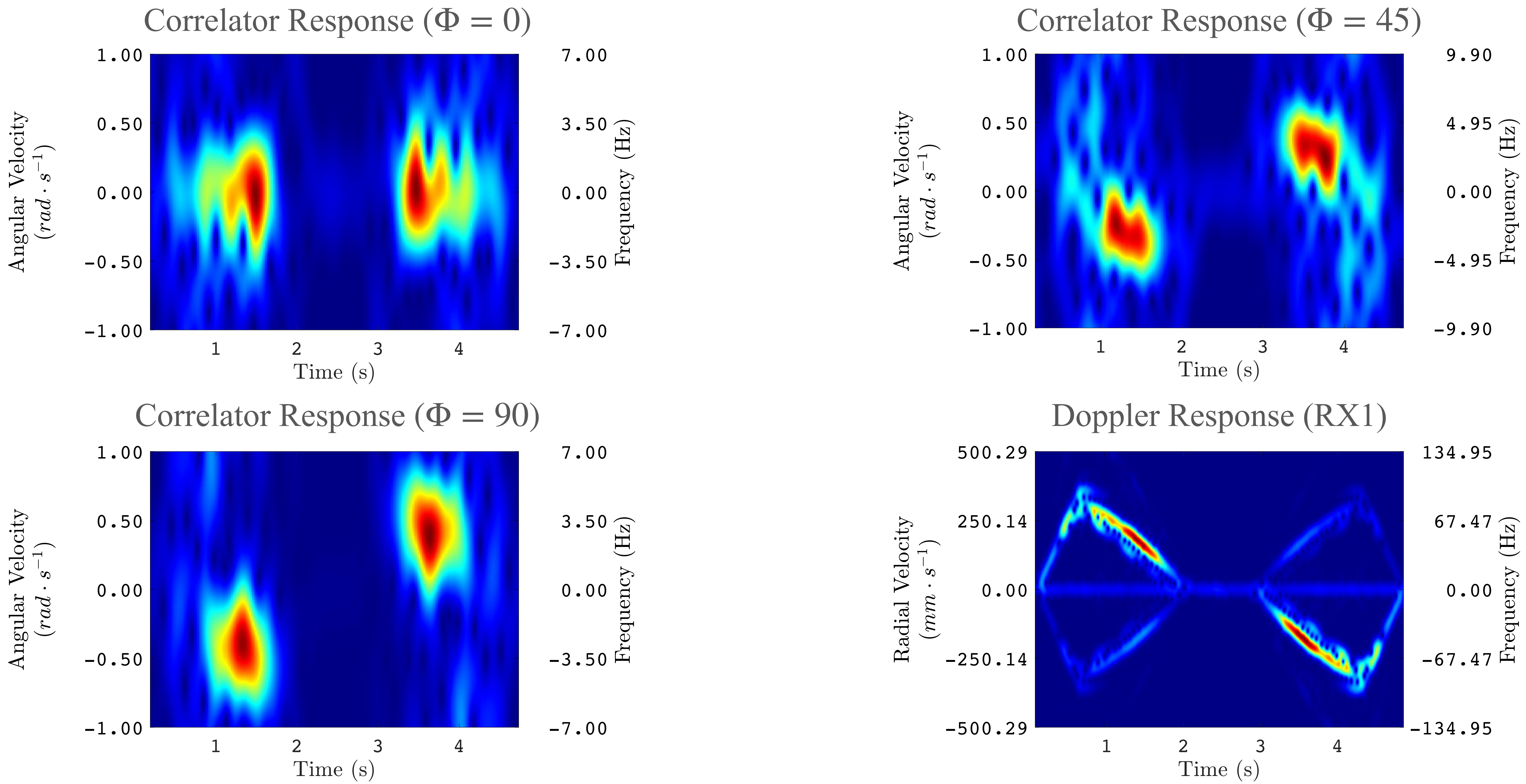




Elevation Estimate Configuration

Experimental Validation - Dual-Axis Continuous-Wave Interferometer

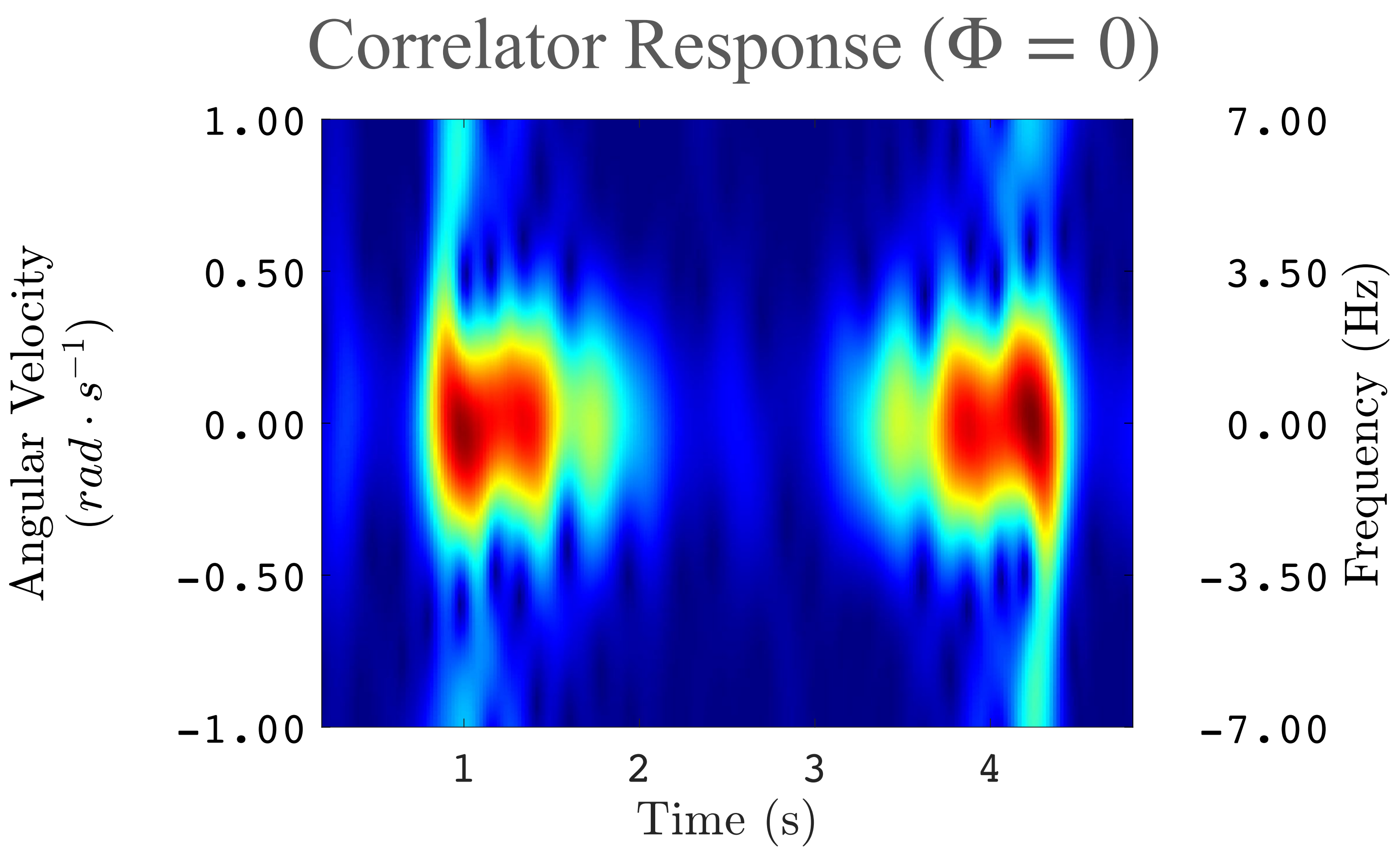
Varying β: $\beta = 30°$; $\phi = 0°$

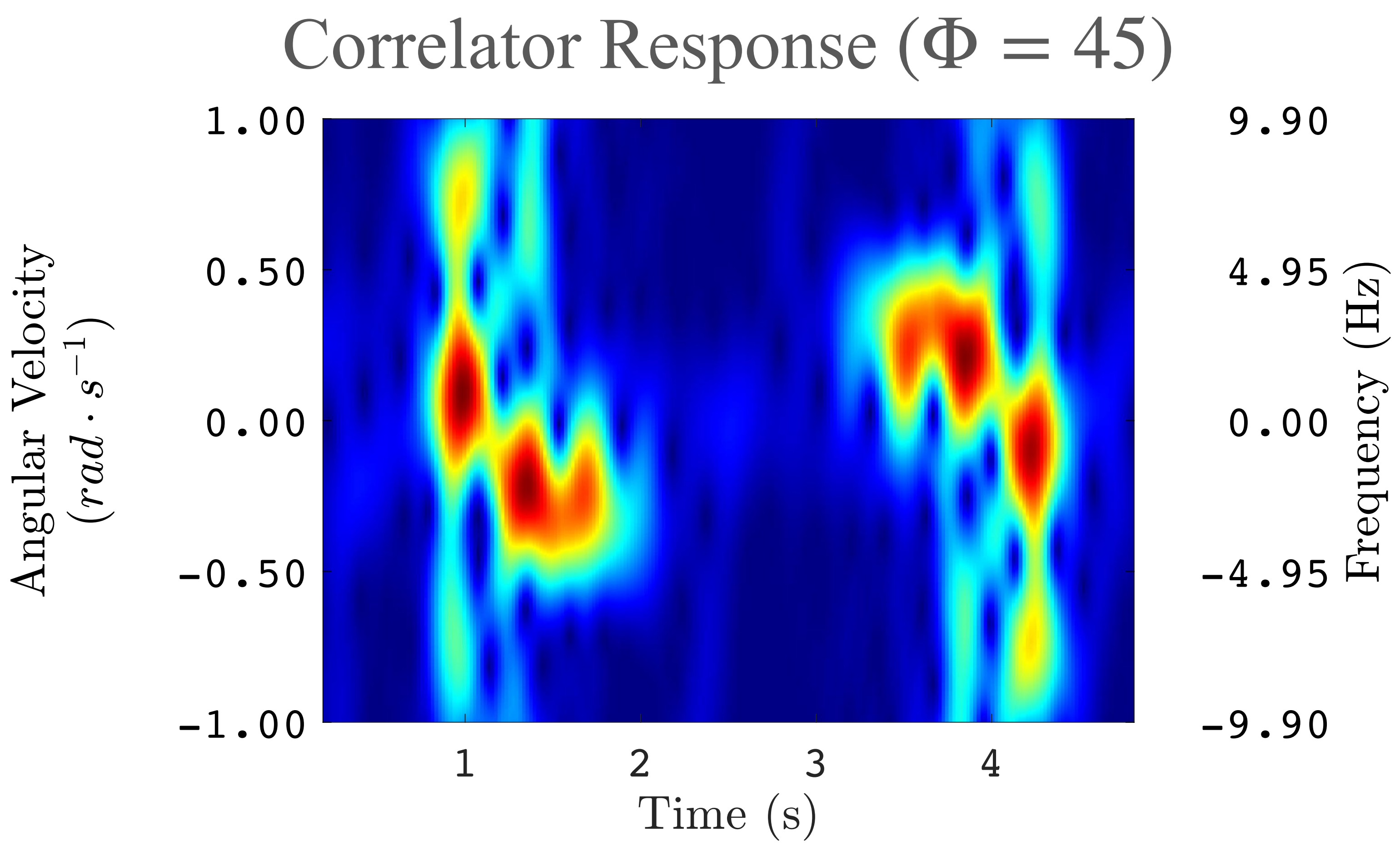


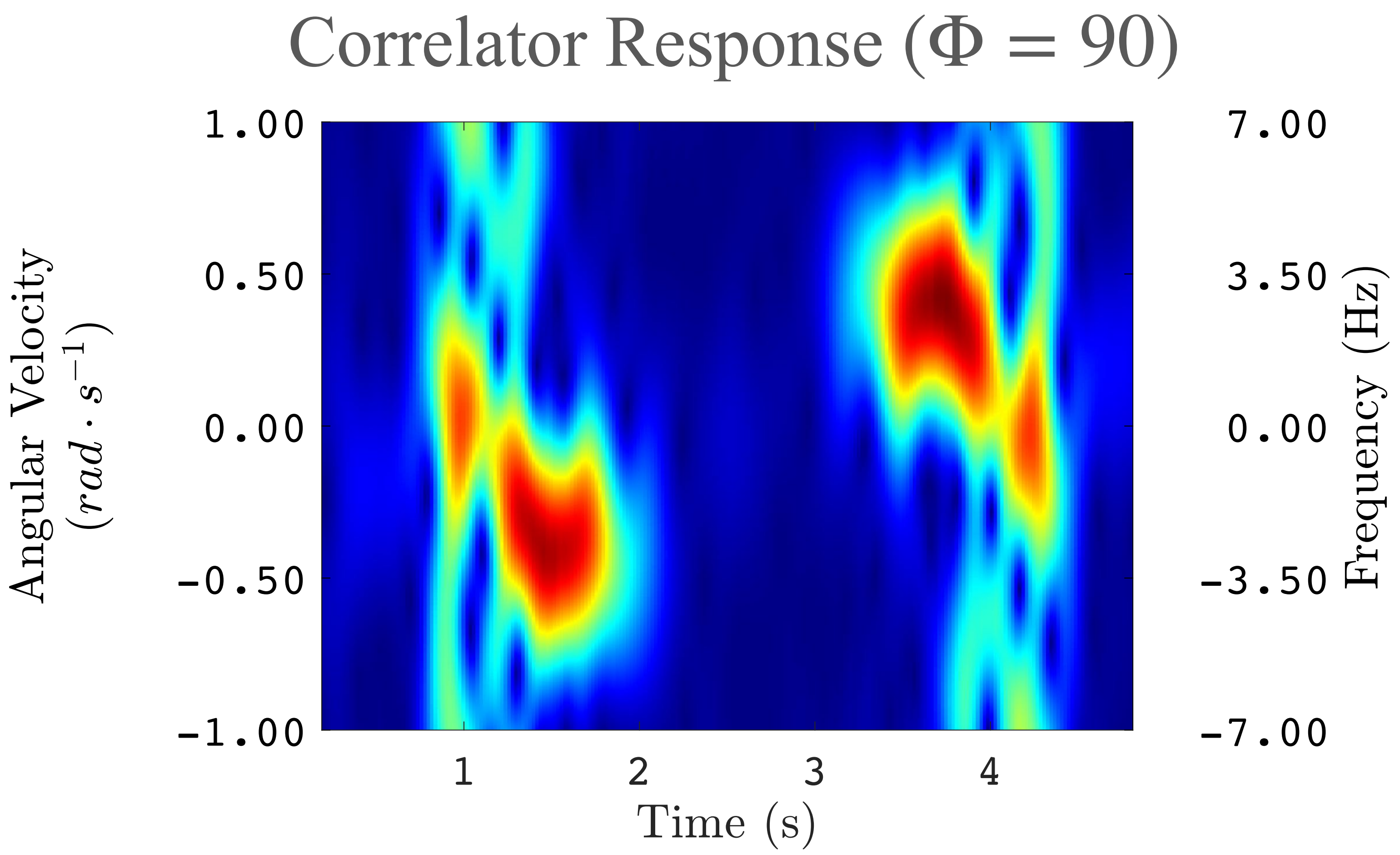
Elevation Estimate Configuration

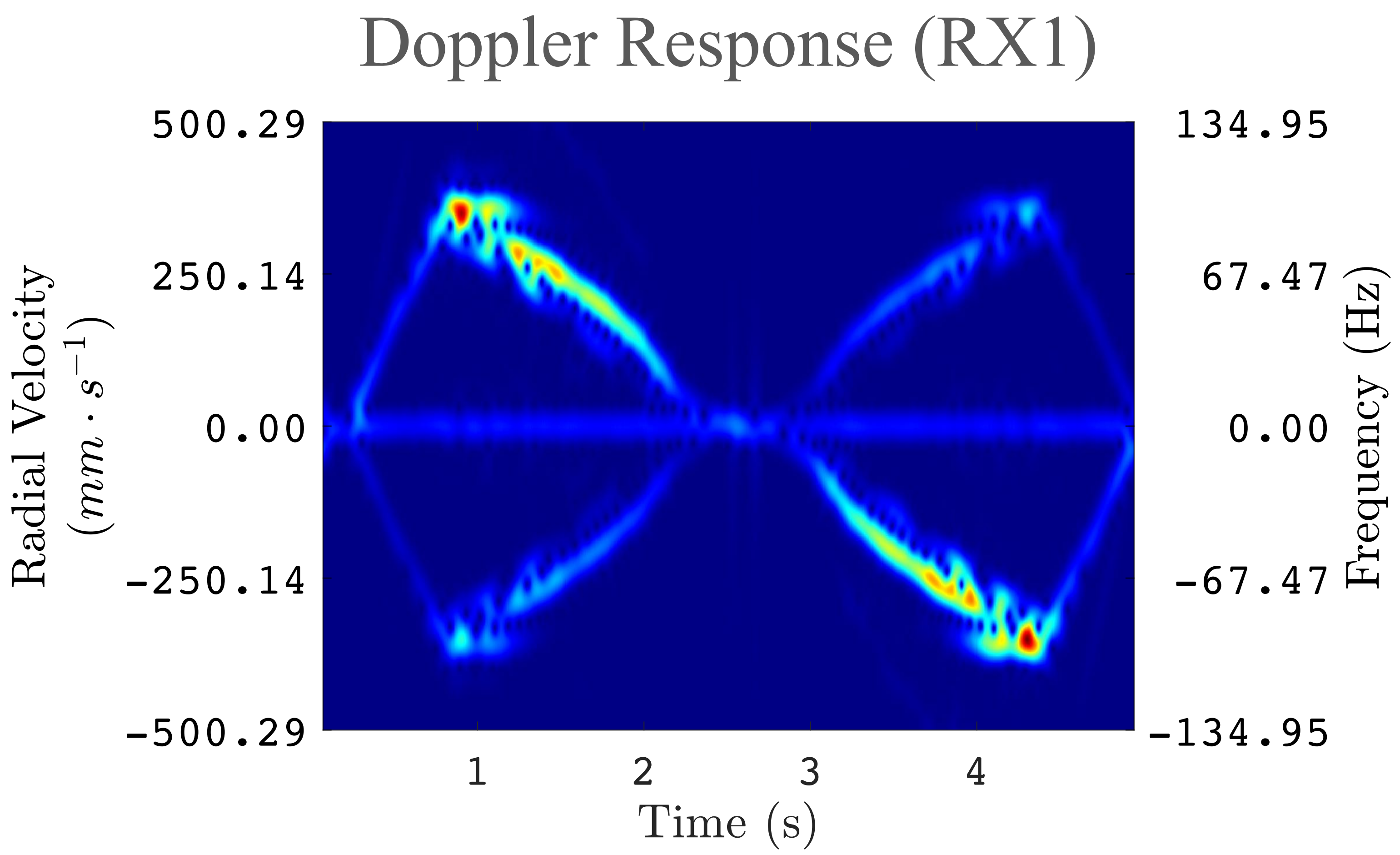
Experimental Validation - Dual-Axis Continuous-Wave Interferometer

Varying β: $\beta = 40°$; $\phi = 0°$





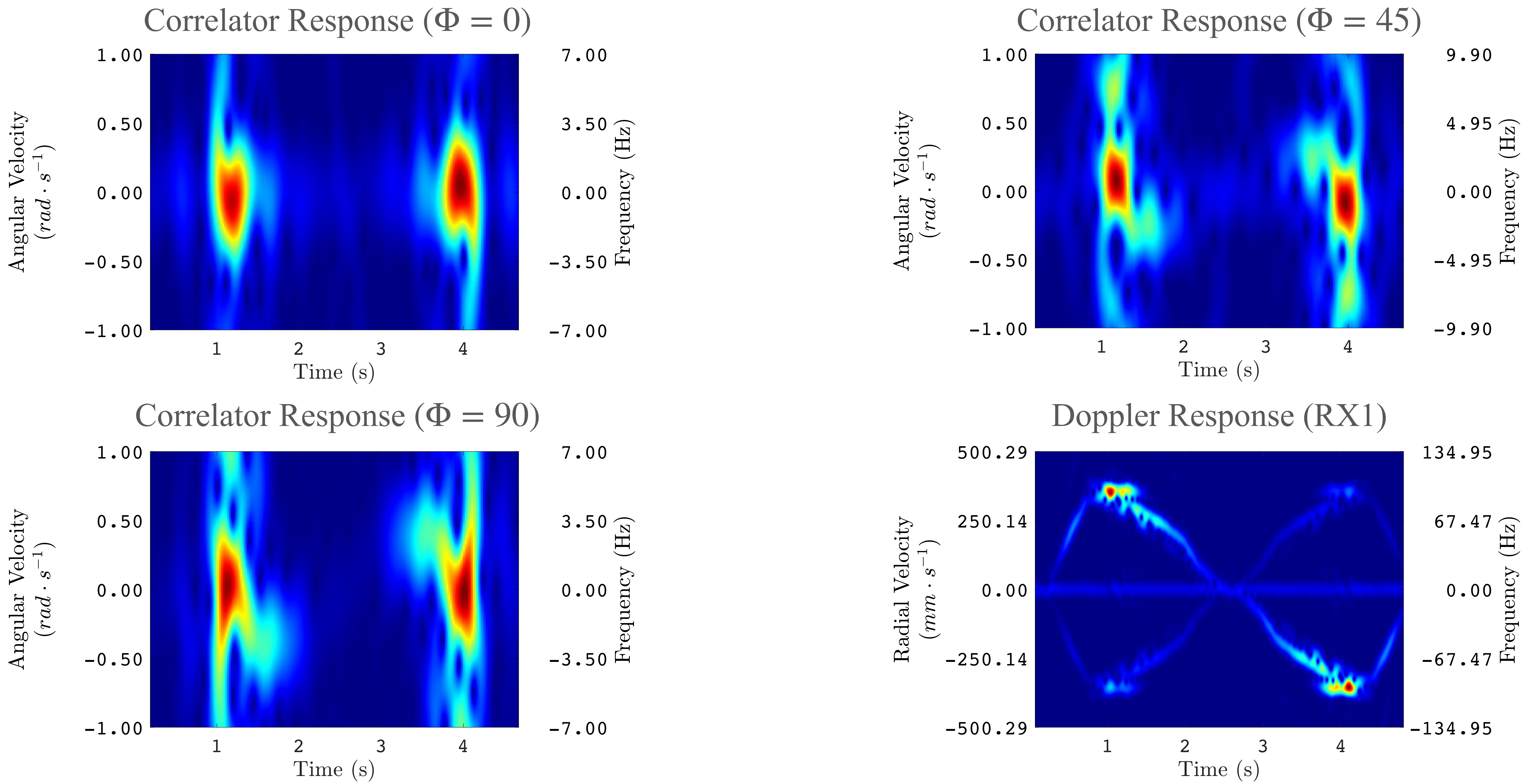




Elevation Estimate Configuration

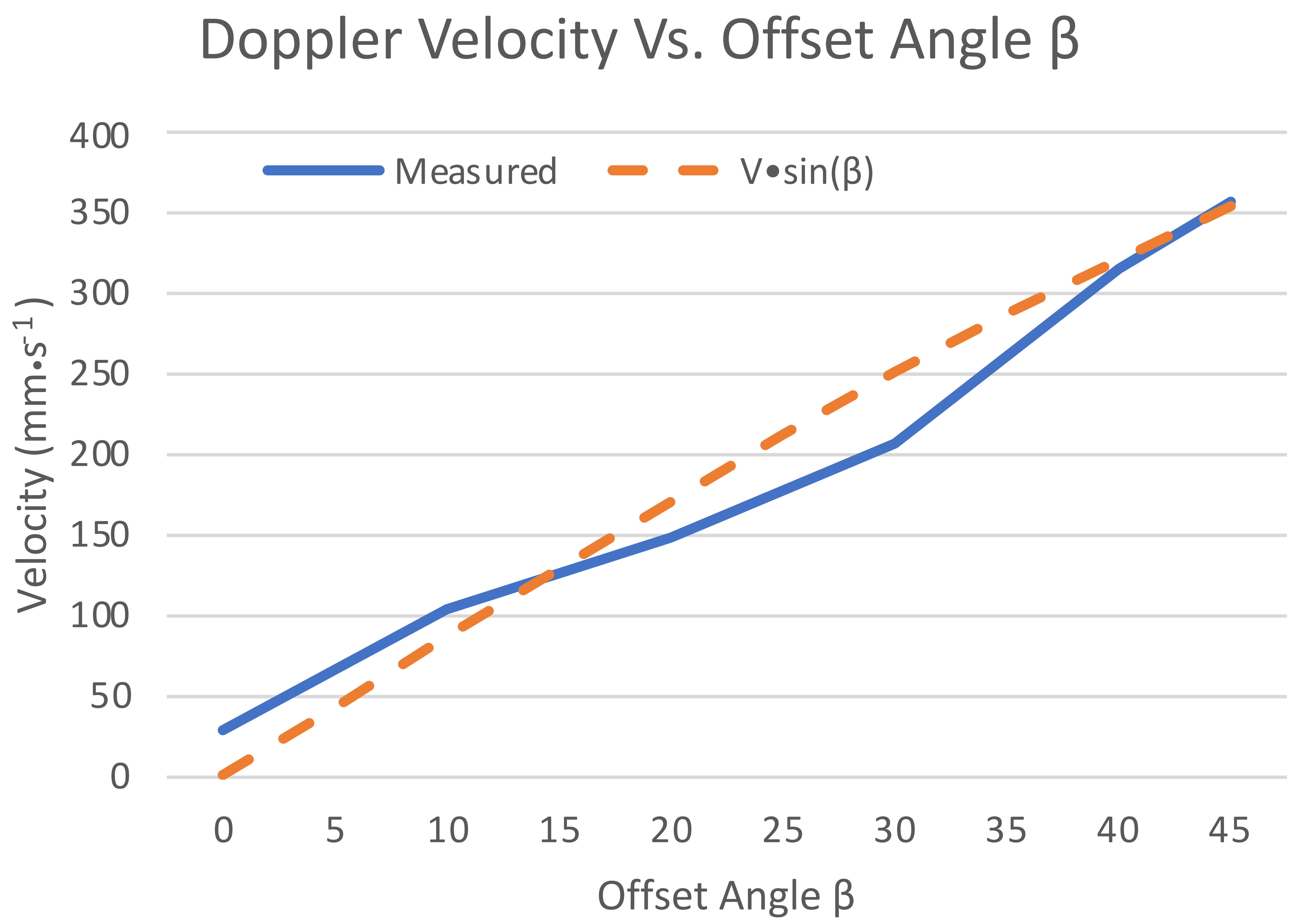
Experimental Validation - Dual-Axis Continuous-Wave Interferometer

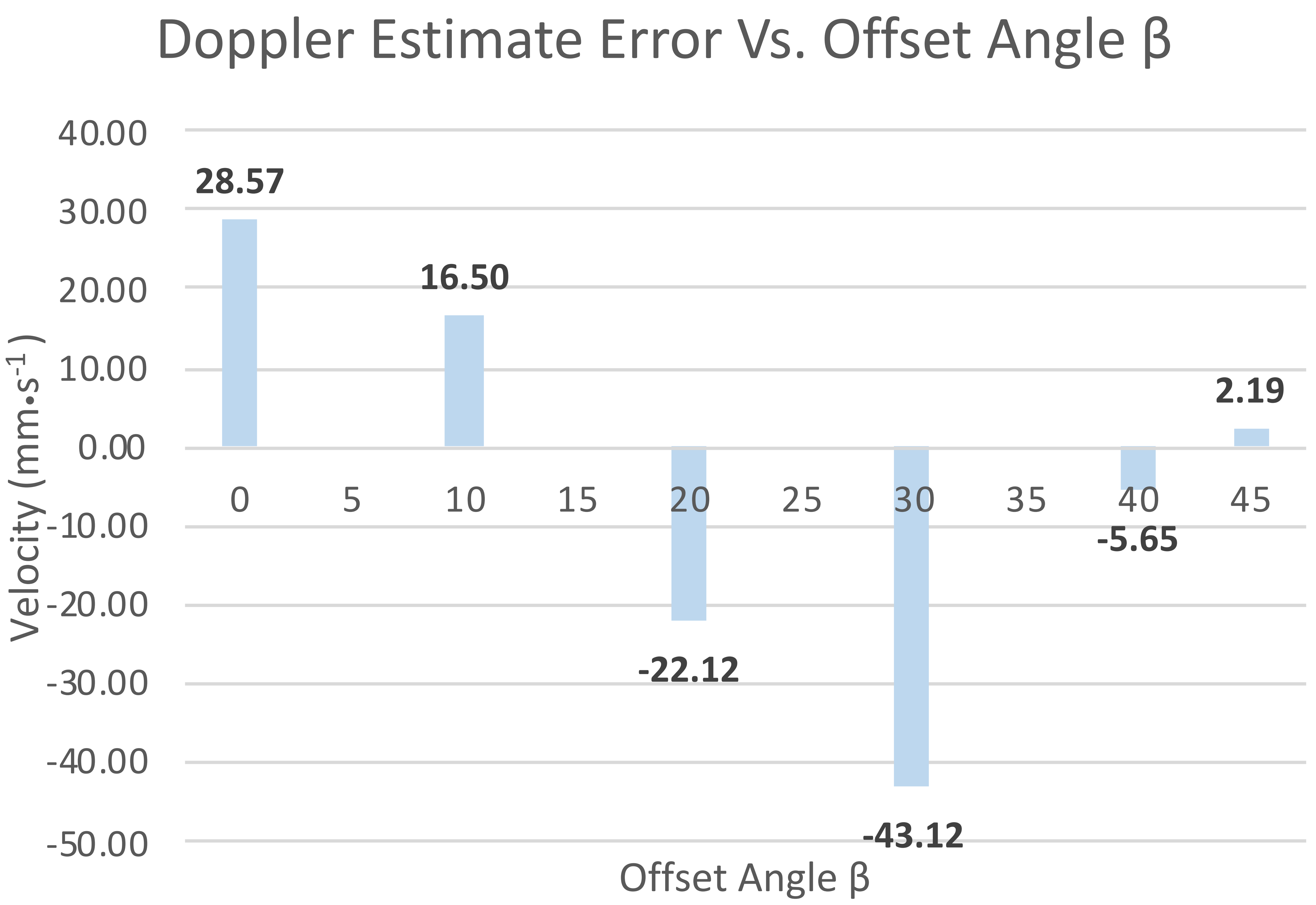
Varying β: $\beta = 45°$; $\phi = 0°$



Varying Elevation - Velocity Estimates

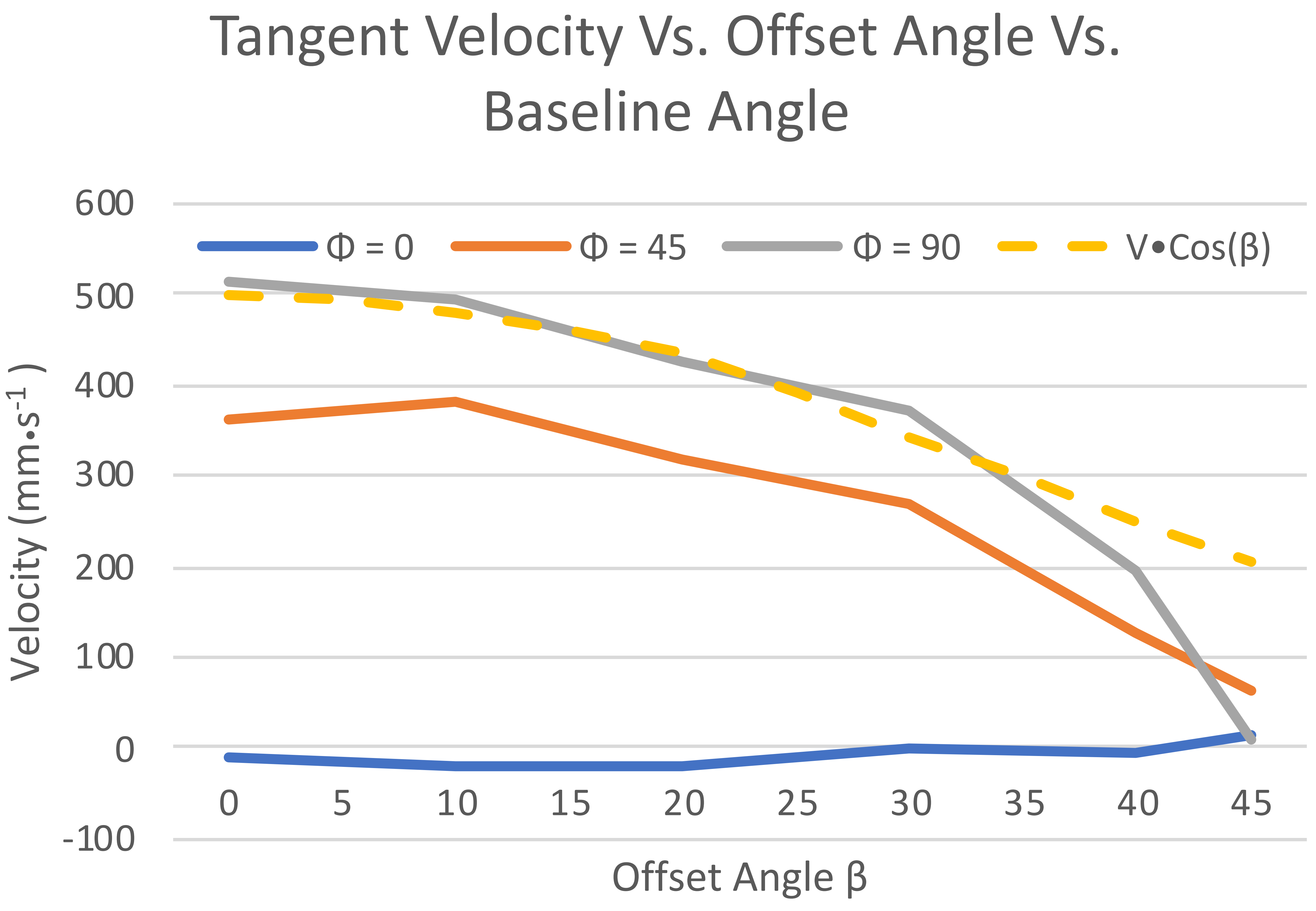
Experimental Validation - Dual-Axis Continuous-Wave Interferometer

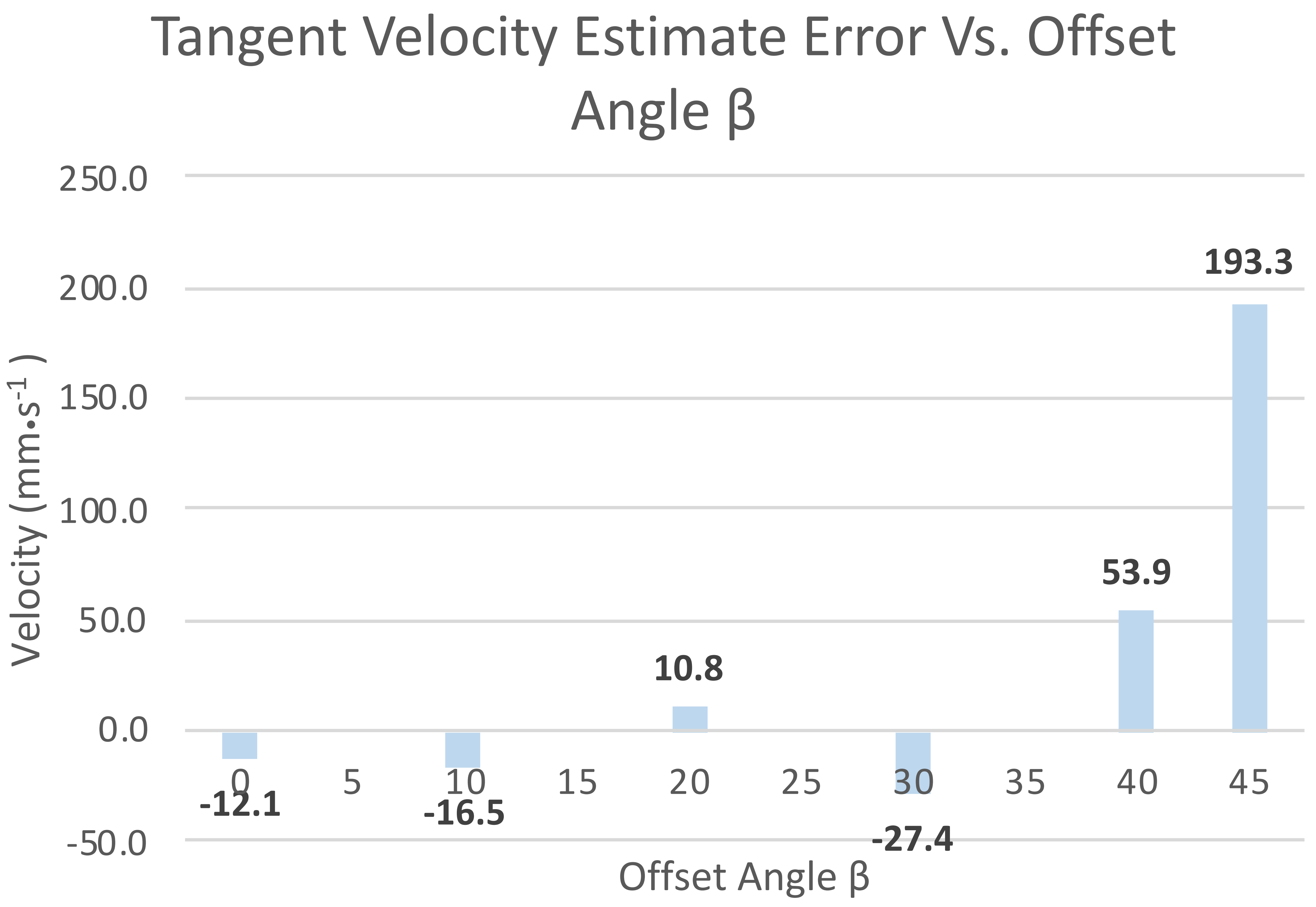




Varying Elevation - Velocity Estimates

Experimental Validation - Dual-Axis Continuous-Wave Interferometer





Varying Elevation - Velocity Estimates

Experimental Validation - Dual-Axis Continuous-Wave Interferometer