



in the presence of  $N$  targets can be found by correlating the received signals at the two antennas of baseline  $m$ ,

$$\begin{aligned} r_{Dm} &= \left\langle r_{d1}^{(m)} \cdot r_{d2}^{*(m)} \right\rangle \\ &= \left\langle \sum_n e^{j2\pi f_0 \tau_{1,n}^{(m)}} \cdot \sum_n e^{-j2\pi f_0 \tau_{2,n}^{(m)}} \right\rangle \end{aligned} \quad (1)$$

where  $r_{d1}^{(m)}$  and  $r_{d2}^{(m)}$  are the baseband signals from the first and second receivers in the  $m$ th baseline,  $\tau_{1,n}^{(m)}$  and  $\tau_{2,n}^{(m)}$  are the round-trip time delays of the signal from the transmitter to the  $n$ th target, back to receivers 1 and 2 of the  $m$ th baseline, and  $\langle \cdot \rangle$  indicates time averaging. Because of the multiplication of all  $N$  targets in (1), the resulting terms will contain differential delays from the same scatterer at each antenna which are denoted  $\tau_{1,n}^{(m)} - \tau_{2,n}^{(m)} = \Delta\tau_{gn}^{(m)}$ , as well as differences between the reflected signal at different targets. As is illustrated in Fig. 1, the time difference between received signals from the same scatterer,  $n$ , at different antennas can be represented as the geometric time delay of a plane-wave impinging on the array, and thus can be represented as  $\Delta\tau_{gn}^{(m)} = \frac{D_m}{c} \sin\theta_n$ . However, there will be an additional  $N(N-1)$  unwanted terms which correspond to the intermodulation between the scattered signals from differing targets at each antenna in the baseline. In general, the angular velocity of the targets determine the instantaneous frequency of (1), which can be found by taking the time derivative of its phase term. The ideal instantaneous frequency from the  $n$ th target, without IMD, is

$$f_n^{(m)} = \frac{d}{dt} \frac{D_m}{c} \sin(\theta_n) = \omega_n D_{\lambda m} \cos(\theta_n) \quad (2)$$

where  $\omega_n$  is the angular velocity of the  $n$ th target,  $D_{\lambda m}$  is the length of the  $m$ th baseline in wavelengths, and the time derivative operates on  $\theta_n = \omega_n t$  [5]. Finally, if  $\theta_n$  is small, the small angle approximation can be used, thus  $\omega_n = f_n^{(m)} / D_{\lambda m}$ .

To remove the  $N(N-1)$  unwanted intermodulation terms, we propose a time-frequency matrix based approach. First, we generate a time-frequency matrix representation of the signal from each baseline by taking the magnitude of the short-time Fourier transform of the signal, then normalize the frequency axis of each baseline matrix by its length in wavelengths  $D_{\lambda m}$ , and finally apply the Hadamard product across all baselines to achieve a *multi-baseline time-frequency product (MBP)* matrix,

$$\mathbf{S}_{MBP} = \mathbf{S}_{1D} \circ \mathbf{S}_{2D} \circ \cdots \circ \mathbf{S}_{MD} \quad (3)$$

where  $\mathbf{S}$  is the magnitude of the short-time Fourier transform. Note that after frequency normalization, all fundamental responses reside at the same frequency. Upon multiplication, the magnitude of the fundamental terms are scaled by  $M$ . The intermodulation products, which do not depend on  $D_{\lambda m}$ , will be scaled arbitrarily and suppressed after multiplication. Note that we choose the baselines to be even multiples of  $D$  for simplicity, however increased baseline diversity will generally improve IMD mitigation by providing greater intermodulation product frequency separation.

### III. SIMULATION

The multi-baseline approach was evaluated in simulation by modeling the responses of two point targets oscillating

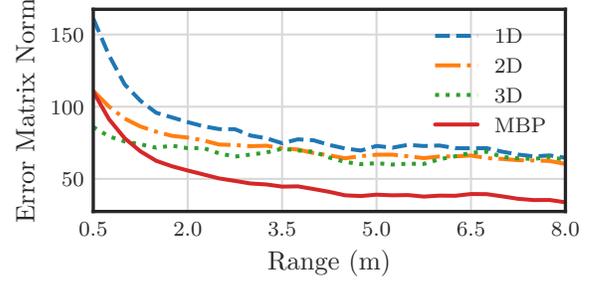


Fig. 2. The norm of the simulated error matrix vs. average target range for each baseline and the MBP. In the near-field, the relationship between the interferometric frequency shift and angular velocity is nonlinear, thus the norm of the error of the MBP is higher.

within the field of view of an array with  $D = 10.71 \lambda$  and  $f_0 = 38$  GHz, as shown in Fig. 1. Each target oscillated sinusoidally, tangentially to the array, while the average target distance was varied from 0.5 m to 8 m. To quantify the reduction of the IMD, we define a matrix norm of the error between the magnitudes of each baseline matrix and the ideal matrix as

$$\|E\| = \frac{1}{N_f N_t} \left\{ \sum_{i=1}^{N_f} \sum_{j=1}^{N_t} [\log(|\mathbf{S}|) - \log(|\mathbf{S}_{gt}|)]^2 \right\} \quad (4)$$

where  $N_f$  is the fast Fourier transform size,  $N_t$  is the number of time samples,  $\mathbf{S}$  is from the individual baselines, or the MBP, and  $\mathbf{S}_{gt}$  is the corresponding ideal ground truth matrix. The ideal matrix is computed by correlating each target with itself only, and thus contains no nonlinear distortion terms. A plot of the error matrix norm vs. average target range is shown in Fig. 2, showing the benefit of the MBP. As the targets move far-field to the array, the linear relationship between angular velocity and interferometric frequency shift improves the performance.

### IV. EXPERIMENTAL EVALUATION

The radar used for this experiment consisted of a continuous-wave (CW) transmitter and three direct-downconversion receivers spaced  $1D$  and  $2D$  apart (shown in Fig. 3) where  $D = 84.6$  mm or  $10.71 \lambda$  at 38 GHz. The transmitter consisted of a 19 GHz local oscillator signal transmitted from a Keysight FieldFox at 5 dBm, split four ways (one for transmit, three for receive), then doubled in frequency and amplified by an Analog Devices (ADI) HMC6787A active upconverter and amplified by a 24.5 dB ADI HMC7229 power amplifier before being transmitted from a 15 dBi 3D printed horn antenna. The three receivers utilized the same 15 dBi antennas followed by ADI HMC1040 23 dB low-noise amplifiers and ADI HMC6789B downconverters. Finally the baseband in-phase and quadrature signals were sampled by an NI USB-6002 DAQ at 4.167 kSps.

The experiment (Fig. 3) consisted of swinging two pendulums made of polystyrene foam balls coated in copper tape and suspended from above at  $R_1 = 1.49$  m and  $R_2 = 1.67$  m using clear plastic wire of differing lengths – this allowed

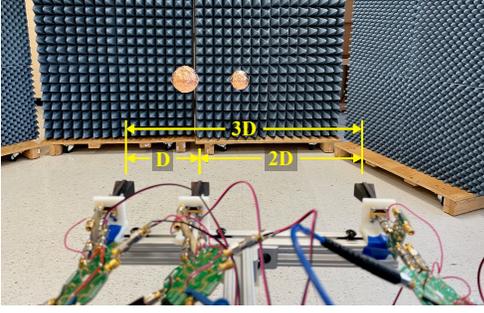


Fig. 3. Experimental measurement configuration. The three baselines of  $1D$ ,  $2D$ , and  $3D$  are in the foreground while the two swinging spherical copper pendulums are suspended by clear wire in the background. The transmitter horn antenna is located below the cross-bar in the center of the array.

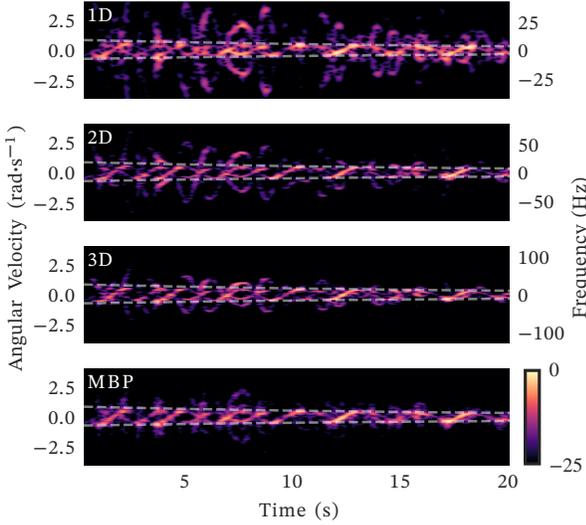


Fig. 4. Measured responses to two oscillating pendulums with 25 dB dynamic range (color scale near MBP applies to all plots). The oscillations of the pendulums start out of phase and end in phase. Dashed lines represent the envelope of the desired oscillations. In the individual baseline measurements, the fundamental frequencies are largely masked by IMD. Upon multiplication the resultant response is principally the fundamental frequencies, validating the multi-baseline approach.

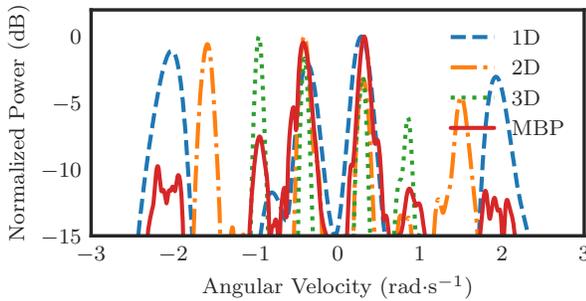


Fig. 5. Time slice of the power spectral density of the  $1D$ ,  $2D$ ,  $3D$ , and MBP measurements taken at  $t = 7.6$  s. A reduction of several dB can be seen in all intermodulation products producing prominent peaks only at the true angular velocity values near  $\pm 0.5$   $\text{rad} \cdot \text{s}^{-1}$ .

the pendulums to oscillate at different frequencies causing the motion to move in and out of relative phase as is seen in

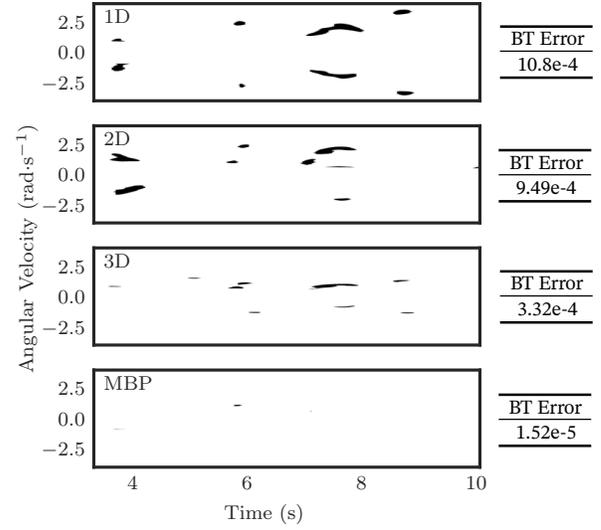


Fig. 6. Binary threshold (BT) error data showing all time-frequency cells outside the pendulum signal envelope with normalized power levels above  $-10$  dB (left), normalized sum of thresholded cells (right). While the resolution improves as  $D$  increases creating thinner intermodulation bands, the intermodulation is still clearly present on each individual baseline.

Fig. 4. The measured time-frequency responses of the  $1D$ ,  $2D$ , and  $3D$  baselines and the MBP are shown in Fig. 4; a one-dimensional time-slice at  $t = 7.6$  s detailing the relative power levels of the intermodulation on each baseline is shown in Fig. 5. Exponentially decaying envelopes (dashed grey lines) were fitted to the signal in the MBP to show the detection of the desired signal envelope. A periodic change in received power is present in both oscillations. These are due to a slight offset of the interferometer to one side of the oscillation which caused more power to be received during the half of the oscillation with positive acceleration due to the high gain antennas; additionally, the rear pendulum experienced minor shadowing by the sphere in the foreground. In the measured case, perfect ground-truth knowledge of the scene is not available, thus another metric had to be defined, thus, to quantify the effect of the performance of the MBP compared to the constituent baselines, a *binary threshold (BT) error* metric was implemented which sums all time-frequency bins outside the oscillation frequency envelope with power levels above  $-10$  dB (Fig. 6), normalized by the sum of the total number of time-frequency bins in the matrix. For the  $1D$ ,  $2D$ , and  $3D$  baselines, errors of  $10.8e-4$ ,  $9.49e-4$ , and  $3.32e-4$  resulted. For the MBP, the error was  $1.52e-5$ , more than an order of magnitude less than any individual baseline.

## V. CONCLUSION

A novel method for the mitigation of nonlinear IMD in correlation interferometers in the presence of multiple dynamic targets has been presented. Through the use of multiple unique baseline distances, the intermodulation products on individual baselines may be significantly attenuated using a simple multiplication operation enabling low-complexity systems to directly measure the angular velocity of multiple targets using low-cost CW radar hardware.

## REFERENCES

- [1] S. S. Blackman, *Multiple-Target Tracking with Radar Applications*. Norwood, MA, USA: Artech House, 1986.
- [2] V. C. Chen, F. Li, S.-S. Ho, and H. Wechsler, "Micro-doppler effect in radar: phenomenon, model, and simulation study," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 42, no. 1, pp. 2–21, 2006.
- [3] Y. Kim and T. Moon, "Human detection and activity classification based on micro-doppler signatures using deep convolutional neural networks," *IEEE Geoscience and Remote Sensing Letters*, vol. 13, no. 1, pp. 8–12, Jan 2016.
- [4] T. Fan, C. Ma, Z. Gu, Q. Lv, J. Chen, D. Ye, J. Huangfu, Y. Sun, C. Li, and L. Ran, "Wireless hand gesture recognition based on continuous-wave doppler radar sensors," *IEEE Transactions on Microwave Theory and Techniques*, vol. 64, no. 11, pp. 4012–4020, 2016.
- [5] J. A. Nanzer, "Millimeter-wave interferometric angular velocity detection," *IEEE Trans. Microw. Theory Techn.*, vol. 58, no. 12, pp. 4128–4136, Dec 2010.
- [6] J. A. Nanzer and K. S. Zilevu, "Dual interferometric-doppler measurements of the radial and angular velocity of humans," *IEEE Trans. Antennas Propag.*, vol. 62, no. 3, pp. 1513–1517, 2014.
- [7] H. Liang, X. Wang, M. S. Greco, and F. Gini, "Enhanced hand gesture recognition using continuous wave interferometric radar," in *2020 IEEE Int. Radar Conf.*, 2020, pp. 226–231.
- [8] J. Nanzer and K. Zilevu, "Distortion mitigation in interferometric angular velocity measurements," *Electron. Lett.*, vol. 50, no. 18, pp. 1316–1318, 2014.
- [9] X. Wang, P. Wang, and V. C. Chen, "Simultaneous measurement of radial and transversal velocities using interferometric radar," *IEEE Trans. Aerosp. Electron. Syst.*, 2019.
- [10] P. Wang, H. Liang, X. Wang, and E. Aboutanios, "Transversal velocity measurement of multiple targets based on spatial interferometric averaging," in *2020 IEEE Int. Radar Conf.* IEEE, 2020, pp. 709–713.
- [11] V. G. Welsby and D. G. Tucker, "Multiplicative receiving arrays," *Journal of the British Institution of Radio Engineers*, vol. 19, no. 6, pp. 369–382, 1959.
- [12] S. Vakalis and J. A. Nanzer, "Millimeter-wave angle estimation of multiple targets using space-time modulation and interferometric antenna arrays," *arXiv preprint arXiv:2008.00356*, 2020.