# A Dual-Axis Interferometric Radar for Instantaneous 2D Angular Velocity Measurement 

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#### Abstract

A complex correlation interferometric radar utilizing two orthogonal baselines to simultaneously measure the angular velocity of a moving target in two dimensions is presented. Measurements were conducted using a 40.5 GHz continuous-wave radar achieving an accuracy of $\mathbf{0 . 5 5}$ to $\mathbf{2 . 5 5 \%}$.


## I. Introduction

Since the advent of the radar, a significant amount of effort has gone into the measurement of an object's motion in space. The ability to measure an object's motion in multiple dimensions allows for one to predict an object's future state. Furthermore, it has been shown that the ability to measure the micro-motion of a target enables remote target type and activity classification [1]. These techniques facilitate many applications in defense and safety systems such as air-traffic control, perimeter security, and collision avoidance systems. Presently, these systems rely on radars which only directly measure instantaneous radial velocity via Doppler frequency shift and must use either mechanical scanning, or large scanning arrays to estimate angular rate over multiple sweeps.

By applying an interferometric processing approach, it has been shown that instantaneous angular velocity can be directly measured using only two continuous-wave (CW) receivers [2], [3]. As an object passes by the aperture of the interferometer, the scattered fields are captured at each antenna, downconverted coherently, then correlated and integrated to produce a frequency that is proportional to the angular velocity of the target at the output of the interferometer. Using this process, it has been shown that both the instantaneous angular velocity and the instantaneous radial velocity can be simultaneously measured using Doppler processing in addition to interferometric processing [4], [5]. Recently, correlation interferometry has been used for target micro-motion measurement and classification in multiple dimensions measuring rotating blades with much higher fidelity than radial micro-Doppler alone [6].

In this work, we present a dual-axis direct-downconversion complex correlation interferometric radar for the direct measurement of the instantaneous motion of a point-scatterer in two dimensions as a step towards full instantaneous threedimensional velocity measurement. By employing two orthogonal receiver baselines, it is possible to determine the angular velocity of a target about two orthogonal spherical axes; a redundant $45^{\circ}$ baseline is included in this work for the purpose of illustrating off-axis motion, but could be used to enhance measurement estimates in future work.


Fig. 1. Simplified radar system schematic (left). Experimental setup (right).


Fig. 2. Positioning of the transmit and receive horns and the measurement coordinate system. The angle $\theta$ is the elevation of the target projected onto the plane formed by the measurement horns and the normal vector of the array. The angle $\Phi$ is measured between the heading of the target and the antenna baseline; e.g. if the two highlighted horns are used to take the measurement, $\Phi=45^{\circ}$.

## II. Measurement System

The physical configuration of the experiment consisted of the multiple baseline interferometric radar and a computercontrolled linear guide with a corner reflector affixed to the sled of the guide as shown in Fig. 1. The sled was set to a command velocity $V$ controlled via stepper motor with a velocity resolution of $0.54 \mathrm{~mm} \cdot \mathrm{~s}^{-1}$. The sled first traveled in the $-\hat{y}$ direction, then returned in the $+\hat{y}$ direction according to the coordinate system depicted in the top left corner of experimental setup in Fig. 1. The radar transmitted at 40.5 GHz from a single 15 dBi horn and received at three 10 dBi horns. Each received signal was amplified, mixed with the carrier, then sampled at 8.003 kHz by a National Instruments USB6002 DAQ. The four antennas were configured in a square with

TABLE I
MEASURED angular velocities $\left(\left|V_{y}\right|=549 \mathrm{~mm} \cdot \mathrm{~s}^{-1}\right)$.

| Angle ( $\Phi$ ) | $v_{\theta}\left(\mathrm{mm} \cdot \mathrm{s}^{-1}\right)$ |  | Error (\%) |  | $v_{\theta}\left(\mathrm{mm} \cdot \mathrm{s}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\hat{V}=-\hat{y}$ |  |  | Error (\%) |  |
| $0^{\circ}$ | -535 | 2.55 |  | 546 | 0.55 |
| $45^{\circ}$ | -392 | 0.98 |  | 385 | 0.82 |
| $90^{\circ}$ | 12 | - |  | -12 | - |

a distance of $D=7 \lambda$ between the centers of each antenna. The antennas were pointed downward towards the linear guide and the frequency responses at the output of the three mixers were recorded as the corner reflector passed under the system. The approximate phase center of the horns was measured to be 71.5 cm from the lowest point inside of the corner reflector.

The angular velocity was measured using the techniques described in [2], [3]. In these works the output of the correlation interferometer is shown to be

$$
\begin{equation*}
r(\theta)=\frac{a_{1} a_{2}}{2} e^{j 2 \pi D_{\lambda} \sin \theta} \tag{1}
\end{equation*}
$$

where $a_{n}$ is the received signal amplitude, $D_{\lambda}$ is the distance between antennas in wavelengths, and $\theta$ is the azimuth angle projected onto the plane defined by a baseline vector $\hat{B}$ and the normal vector of the array shown in Fig. 2. The instantaneous frequency at the output of the interferometer can be found by taking the time derivative of the phase term in (1). Thus,

$$
\begin{equation*}
f_{s}=\omega D_{\lambda} \cos \omega t \tag{2}
\end{equation*}
$$

where $\omega=\frac{d \theta}{d t}$. Due to the narrow beam-width of the antennas used, the cosine term can be neglected as is noted in [2].

Finally, the angular velocity can be solved for by using the relation $\omega=v_{\theta} / R$. In the general case, the measured angular velocity is described by the velocity vector projected onto the plane of the antenna baseline, $\operatorname{Proj}_{\hat{B}} \vec{V}=V \cos \Phi$, where $\hat{B}$ is the unit vector along the antenna baseline. This is due to the fact that the interferometer only measures the component of the angular velocity parallel with its baseline, which leads to the equation for the expected angular velocity measured by the interferometer on a given an angle $\Phi$ between $\hat{V}$ and $\hat{B}$,

$$
\begin{equation*}
v_{\theta}(\Phi)=\frac{f_{s} \lambda R}{D} \cos \Phi \tag{3}
\end{equation*}
$$

This is exploited to directly measure instantaneous $\vec{v}_{\theta x}$ and $\vec{v}_{\theta y}$, the velocity vectors tangential to the interferometer, by choosing orthogonal baselines along the $\hat{x}$ and $\hat{y}$ axes.

## III. Experimental Results

Multi-axis measurements of the instantaneous angular velocities for a target moving at $549 \mathrm{~mm} \cdot \mathrm{~s}^{-1}$ are presented in Fig. 3. To produce each spectrogram, fast Fourier transforms (FFT) of size $2^{10}$ were used with a Hamming window of length 350 ms ; a $95 \%$ data overlap was used between each FFT. Note that due to the square layout of the receivers, the $\Phi=45^{\circ}$ baseline is $\sqrt{2}$ times longer than the two orthogonal baselines resulting in a frequency response which is $\sqrt{2}$ times


Fig. 3. Interferometric angular velocity measurements for a sled command velocity of $V=549 \mathrm{~mm} \cdot \mathrm{~s}^{-1}$.
larger than the orthogonal baseline pairs according to (2). Also note that the interferometric response drops in the center as the target passes broadside. This is due to the fact that directdownconversion systems with a limited time-window require at least a small Doppler shift such that the differential phases manifest in the response.

The peak angular velocities measured for each baseline are summarized in Table I. The peak velocities for the measurements were estimated by first finding frequency of the peak power of each FFT slice in the spectrogram and then averaging all of the peaks within $40 \%$ of the maximum peak power for each pass of the corner reflector. These results show that instantaneous angular velocity can be directly measured with a two-axis system, in this work achieving an overall accuracy of 0.55 to $2.55 \%$ from the ground-truth commanded velocity.

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