Student Paper Finalist



Tu2C-3

# High Accuracy Wireless Time-Frequency Transfer for Distributed Phased Array Beamforming

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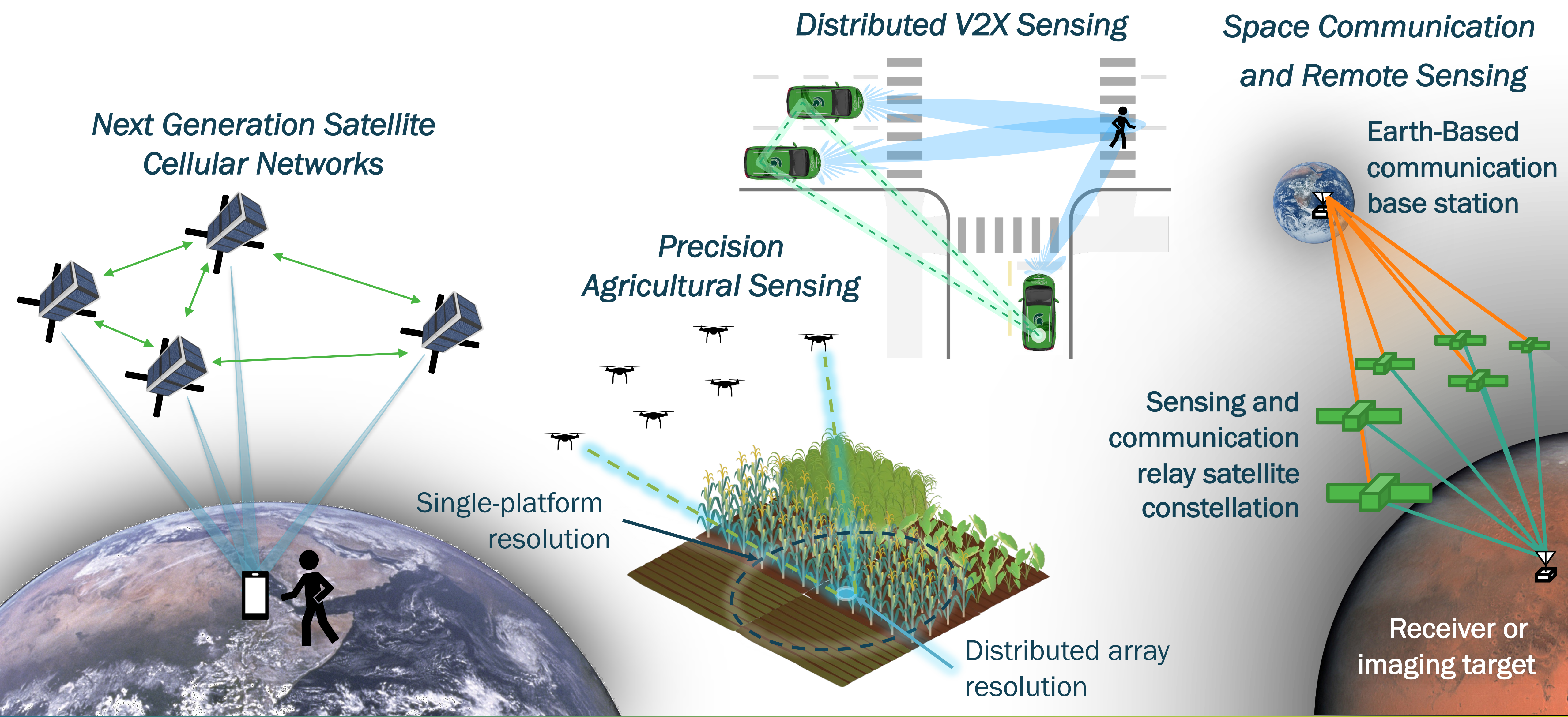
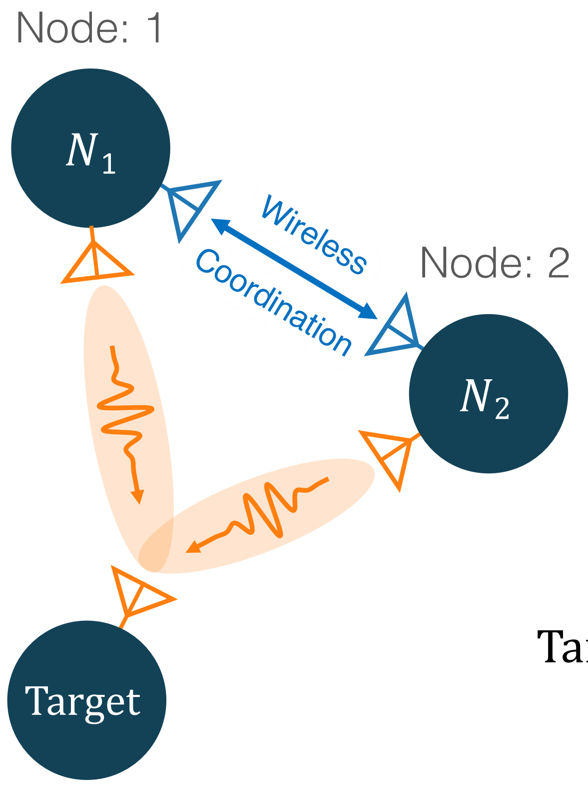
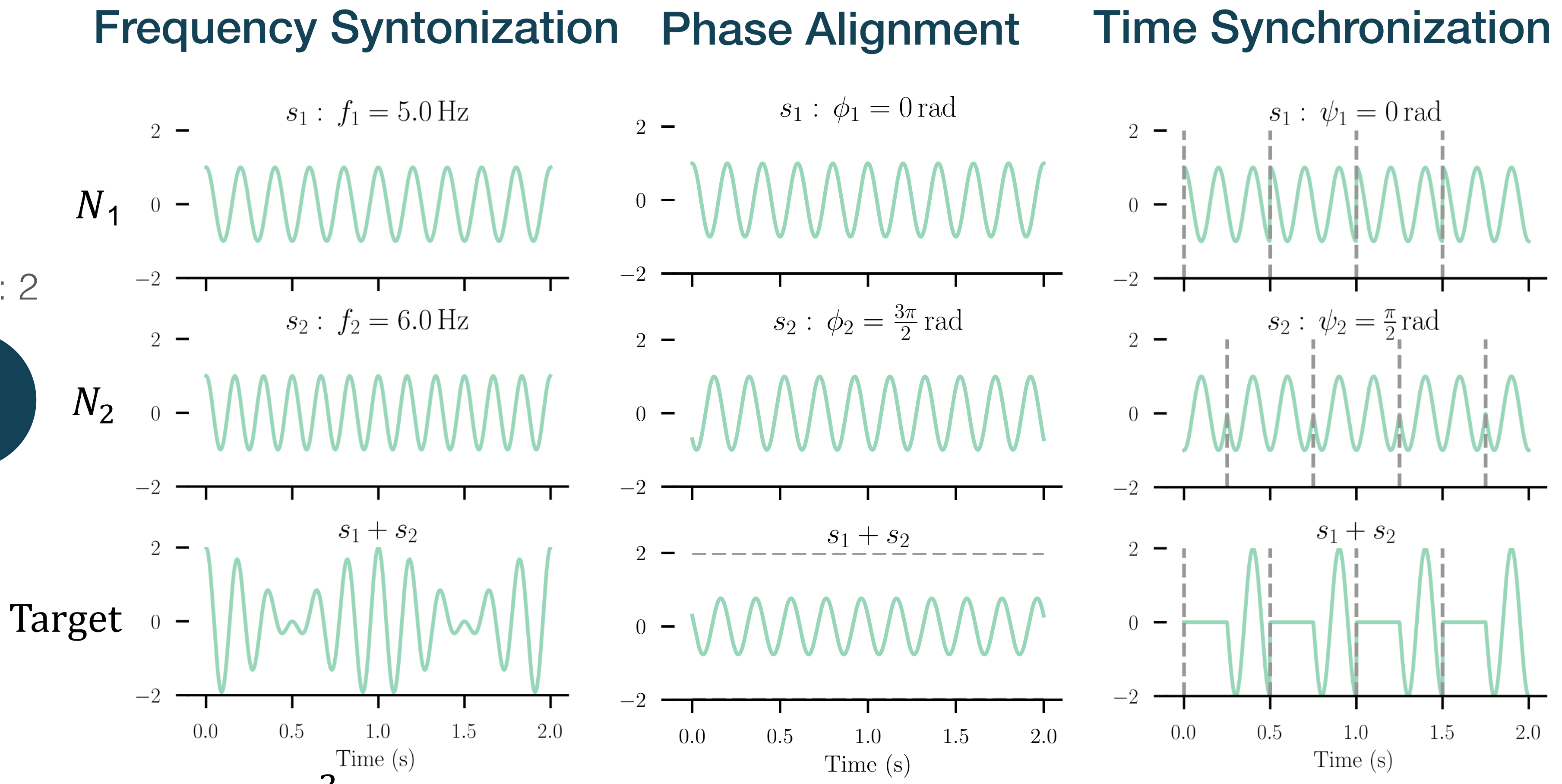
# Distributed Phased Arrays



# Distributed Array Synchronization




$$s_1 + s_2 = \sum_{n=1}^{2} \alpha_n(t - \delta t_n) \exp\{j[2\pi(f + \delta f_n) + \phi_n]\}$$

# Distributed Array Performance



Probability of coherent gain:

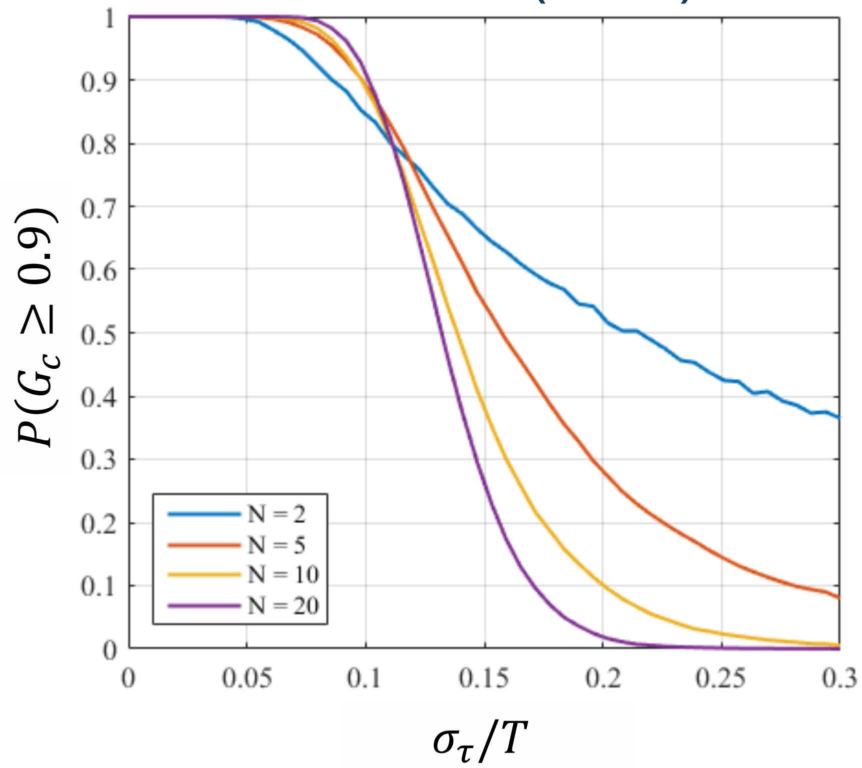
$$P(G_c \geq X)$$

where

$$G_c = \frac{|s_r s_r^*|}{|s_i s_i^*|}$$

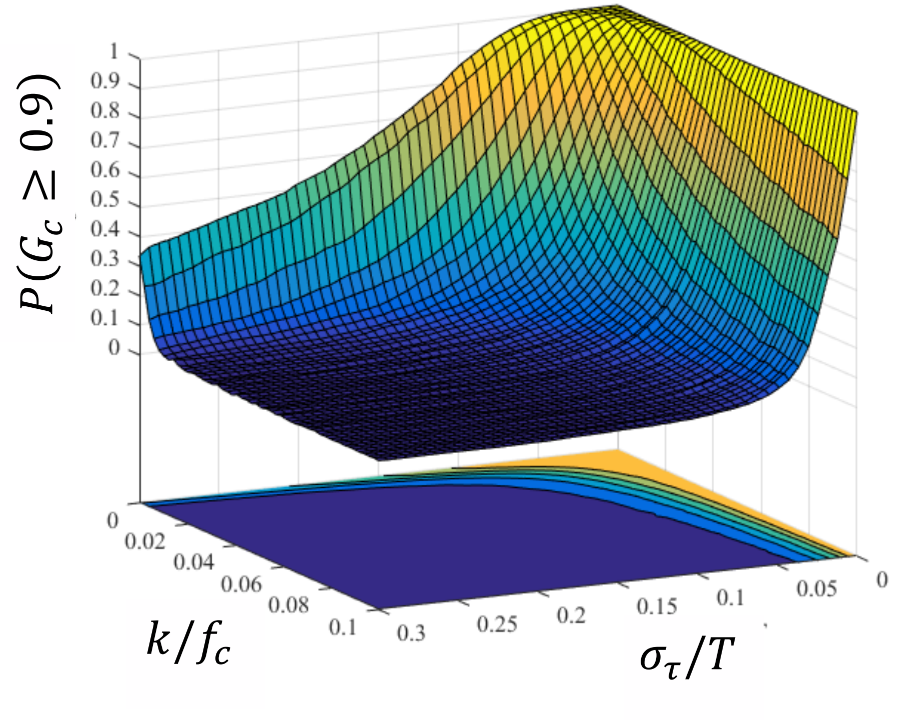
- $s_r$: received signal
- $s_i$: ideal signal

## Barker Code (13-bit)




Timing error <10% pulse duration

## Linear Frequency Modulated




Modulation requires stricter timing

[1] J. A. Nanzer, R. L. Schmid, T. M. Comberiate and J. E. Hodkin, "Open-Loop Coherent Distributed Arrays," in IEEE Transactions on Microwave Theory and Techniques, vol. 65, no. 5, pp. 1662-1672, May 2017, doi: 10.1109/TMTT.2016.2637899.
[2] P. Chatterjee and J. A. Nanzer, "Effects of time alignment errors in coherent distributed radar," in Proc. IEEE Radar Conf. (RadarConf), Apr. 2018, pp. 0727–0731.

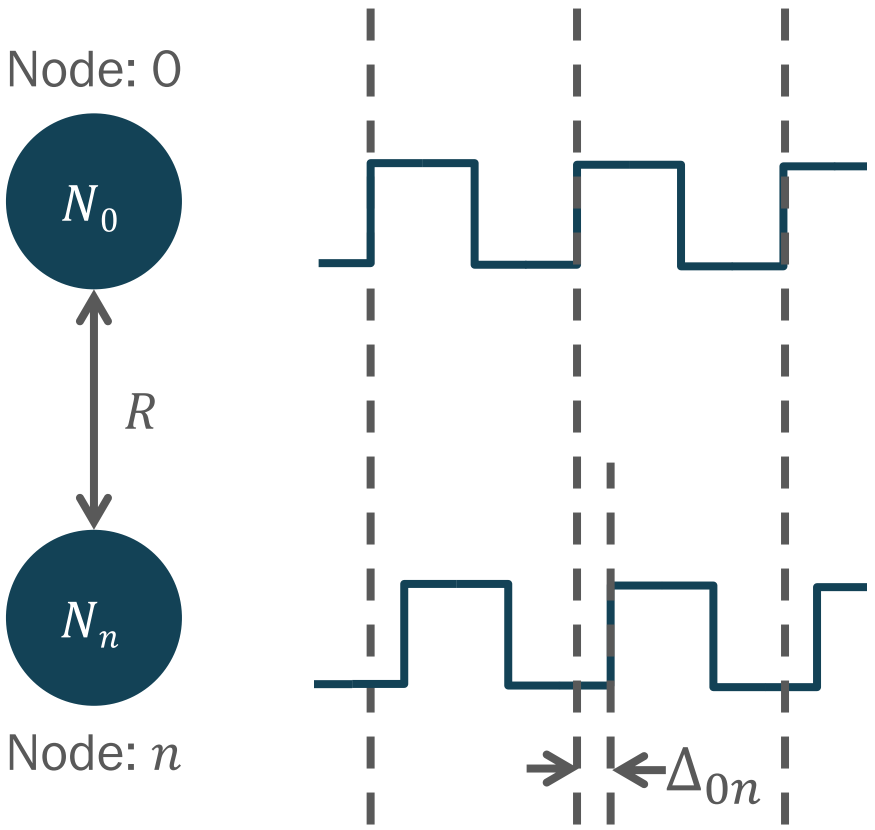
# System Time Model

- Local time at node $n$:

$$T_n(t) = t + \delta_n(t) + \nu_n(t)$$

  - $t$ : true time
  - $\delta_n(t)$: time-varying offset from global true time
  - $\nu_n(t)$: other zero-mean noise sources
  - $\Delta_{0n}(t) = T_0(t) - T_n(t)$
- Goal: estimate and compensate for $\Delta_{0n}$

Relative Clock Alignment



# Achieving Synchronization



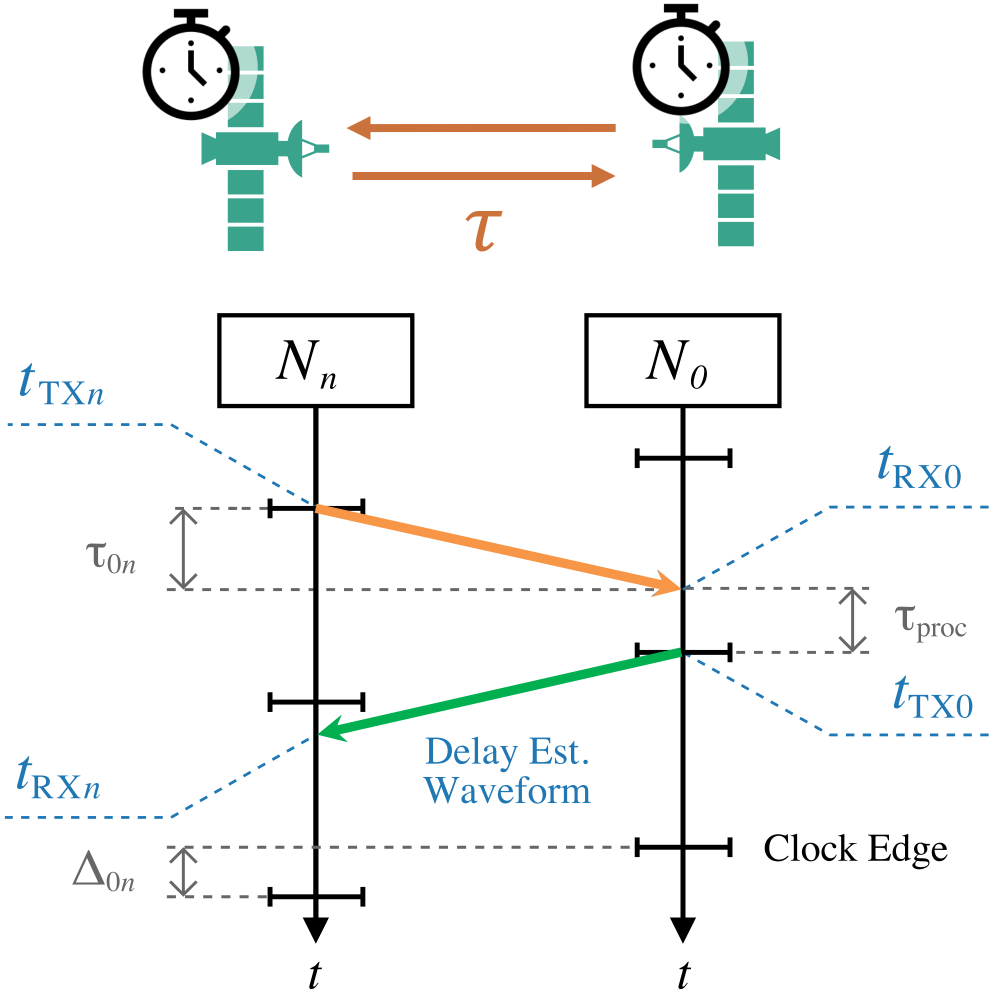
**Two-Way Time Synchronization**

- *Assumptions:*
  - Link is quasi-static and reciprocal during the synchronization epoch

- Timing skew estimate:

$$\Delta_{0n} = \frac{(T_{\mathrm{RX0}} - T_{\mathrm{TX}n}) - (T_{\mathrm{RX}n} - T_{\mathrm{TX0}})}{2}$$

For compactness of notation: $T_m(t_{\mathrm{TX}n}) = T_{\mathrm{TX}n}$



# High Accuracy Delay Estimation



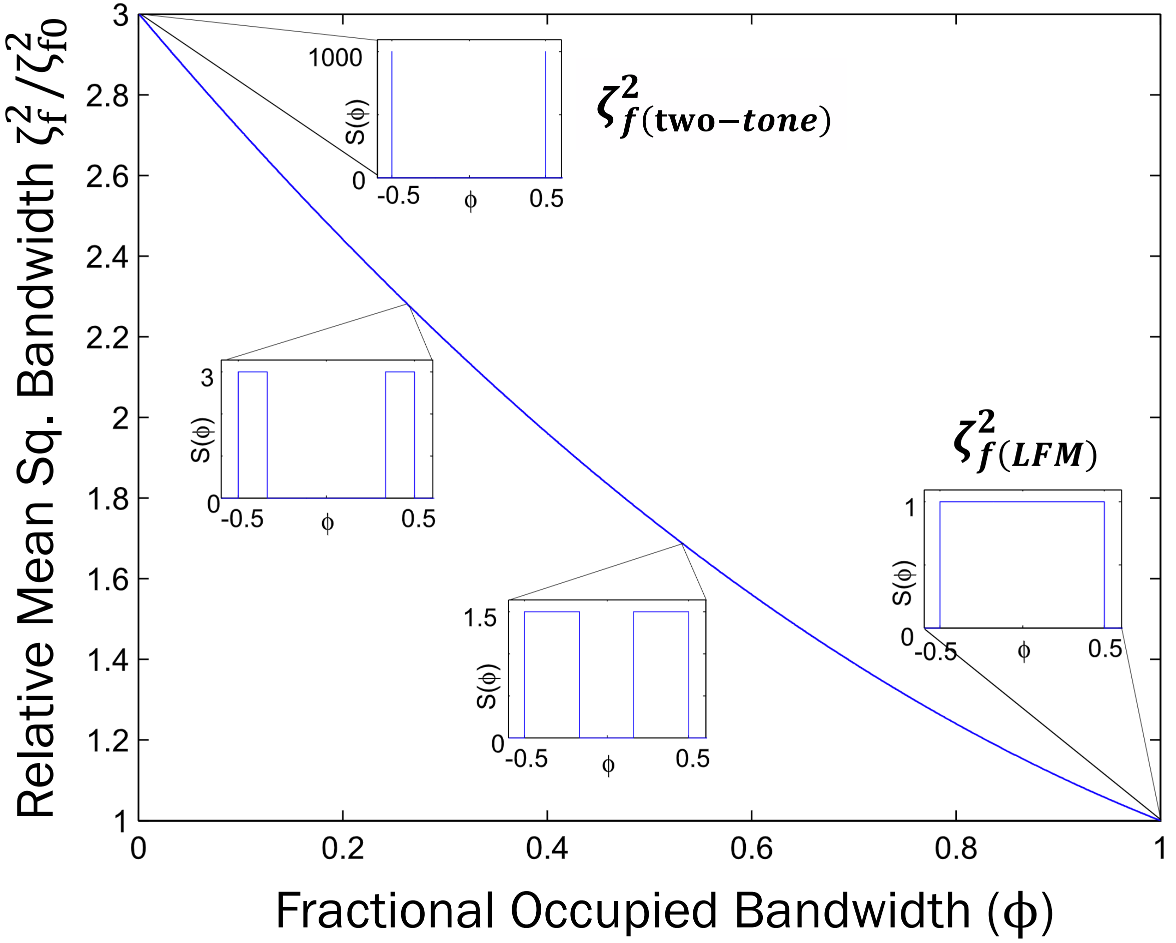
- The delay accuracy lower bound (CRLB) for time is given by

$$\mathrm{var}(\hat{\tau} - \tau) \geq \frac{1}{2\zeta_f^2} \cdot \frac{N_0}{E_s}$$

  - $\zeta_f^2$: mean-squared bandwidth
  - $N_0$: noise power spectral density
  - $E_s$: signal energy

$$\frac{E_s}{N_0} = \tau_p \cdot \mathrm{SNR} \cdot \mathrm{NBW}$$

  - $\tau_p$: integration time
  - SNR: signal-to-noise ratio
  - NBW: noise bandwidth




[3] J. A. Nanzer and M. D. Sharp, “On the Estimation of Angle Rate in Radar,” *IEEE T Antenn Propag*, vol. 65, no. 3, pp. 1339–1348, 2017, doi: 10.1109/tap.2016.2645785.

# High Accuracy Delay Estimation

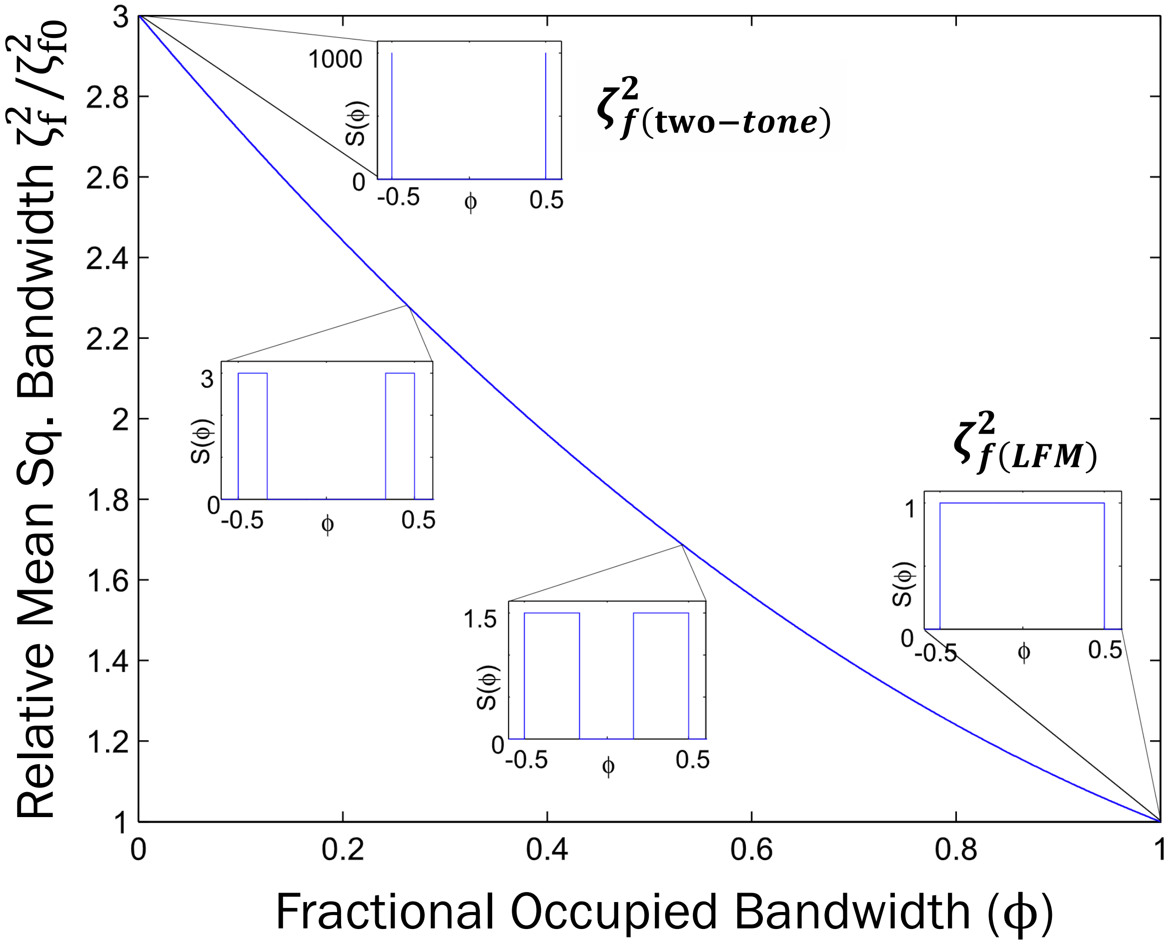


$$\mathrm{var}(\hat{\tau} - \tau) \geq \frac{1}{2\zeta_f^2} \cdot \frac{N_0}{E_s}$$

- For constant-SNR, maximizing $\zeta_f^2$ will yield improved delay estimation

$$\zeta_f^2 = \int_{-\infty}^{\infty} (2\pi f)^2 |G(f)|^2 df$$

- $\zeta_{f(LFM)}^2 = (\pi \cdot \mathrm{BW})^2 / 3$
- $\zeta_{f(\mathrm{two-tone})}^2 = (\pi \cdot \mathrm{BW})^2$

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# Delay Estimation Refinement

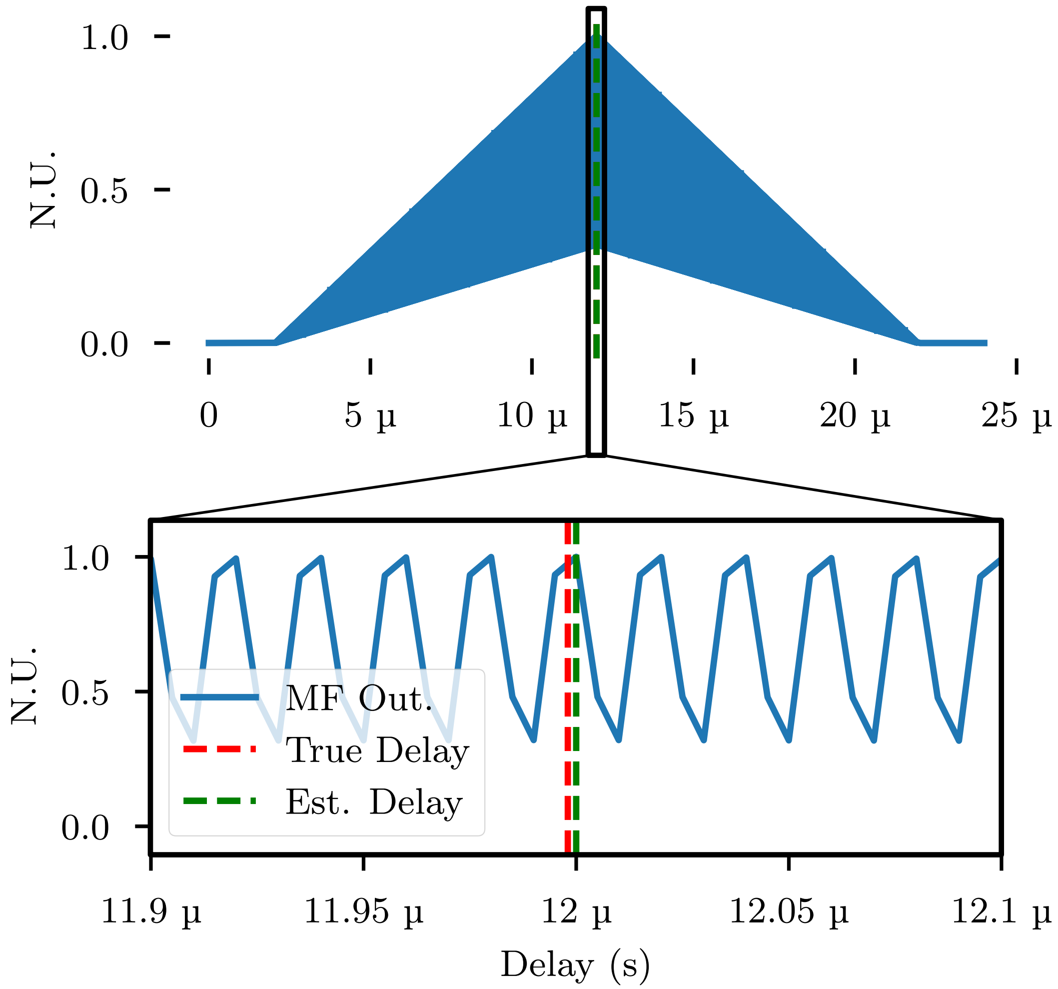


- Discrete matched filter (MF) used in initial time delay estimate

$$s_{\mathrm{MF}}[n] = s_{\mathrm{RX}}[n] \circledast s_{\mathrm{TX}}^*[-n]$$
$$= \mathcal{F}^{-1}\{S_{\mathrm{RX}} S_{\mathrm{TX}}^*\}$$

- Two-tone matched filter waveform is highly ambiguous

- High SNR or narrow-band pulse required to disambiguate peaks



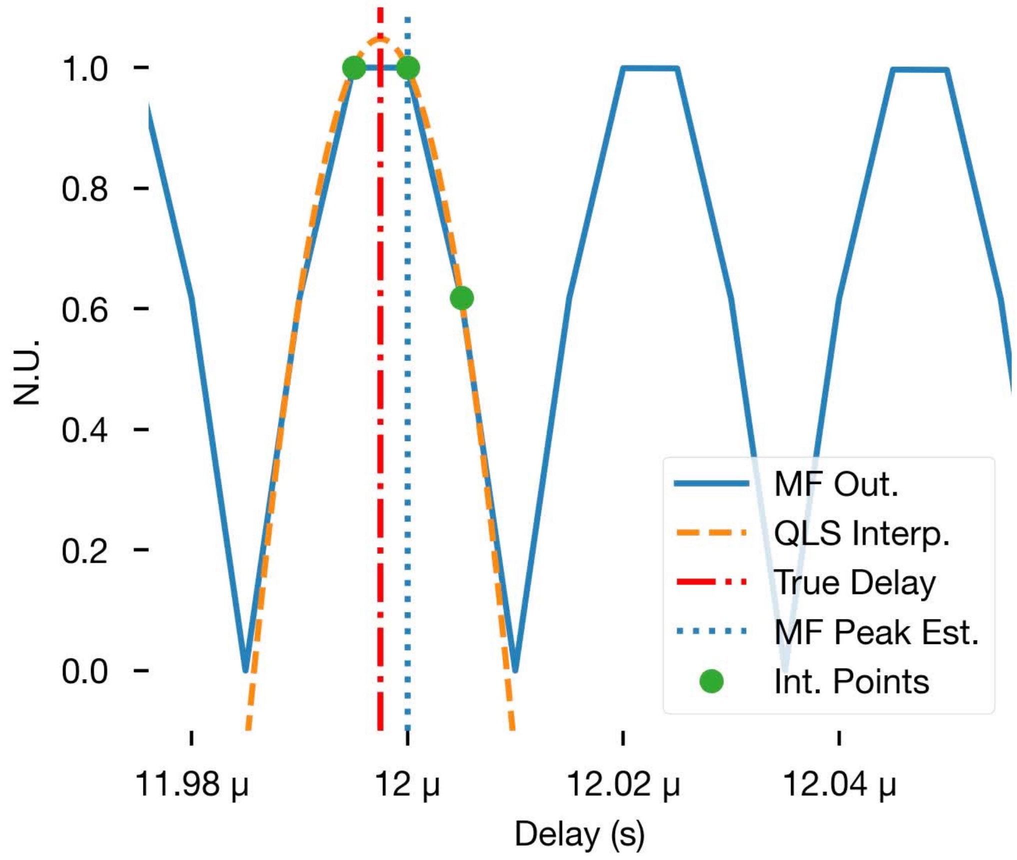
# Delay Estimation Refinement

- MF causes estimator bias due to time discretization
- Refinement of MF obtained using Quadratic Least Squares (QLS) fitting to find true delay based on three sample points

$$\hat{\tau} = \frac{T_s}{2} \frac{s_{\mathrm{MF}}[n_{\max} - 1] - s_{\mathrm{MF}}[n_{\max} + 1]}{s_{\mathrm{MF}}[n_{\max} - 1] - 2s_{\mathrm{MF}}[n_{\max}] + s_{\mathrm{MF}}[n_{\max} + 1]}$$

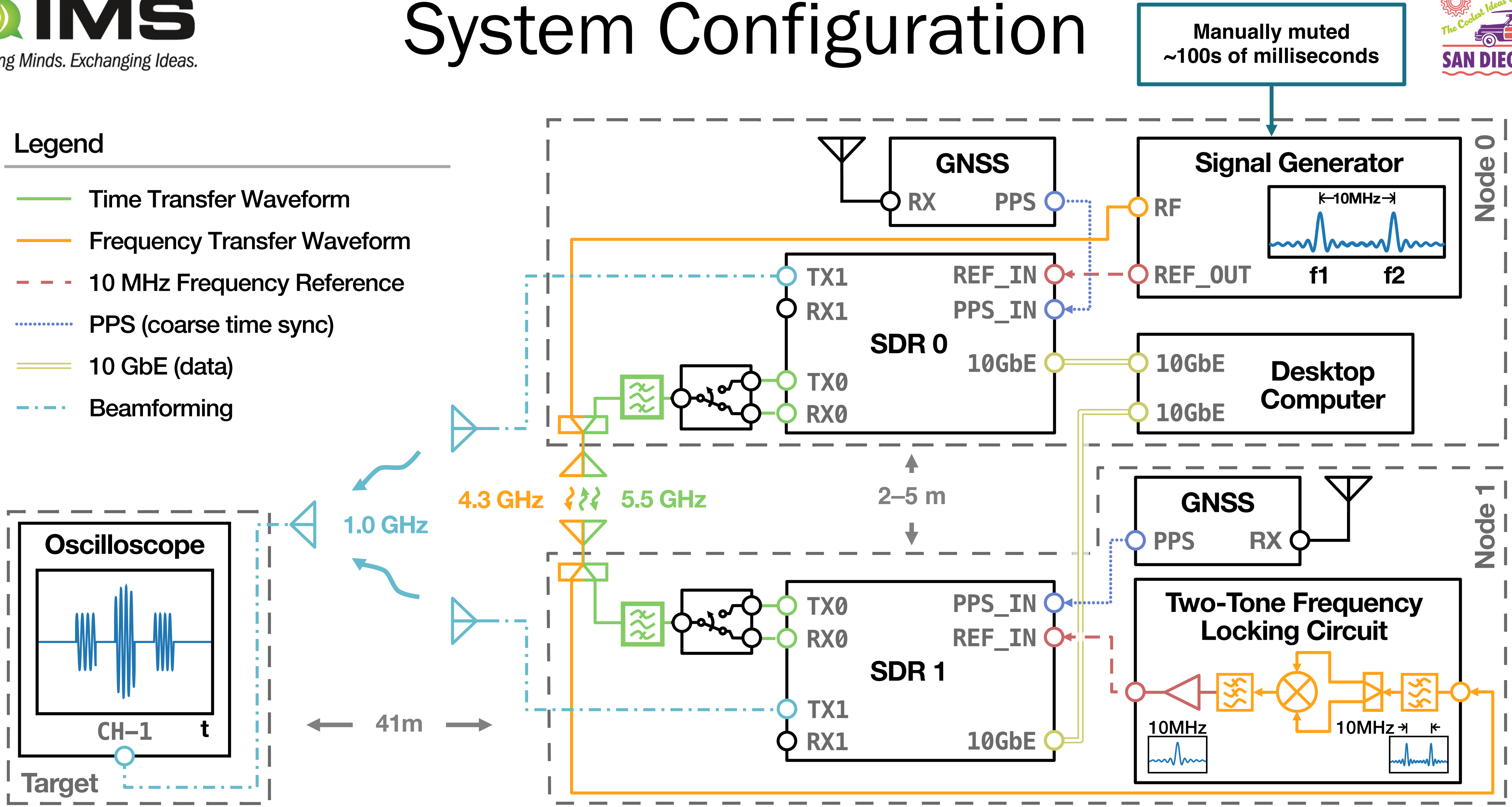
where

$$n_{\max} = \underset{n}{\operatorname{argmax}}\{s_{\mathrm{MF}}[n]\}$$



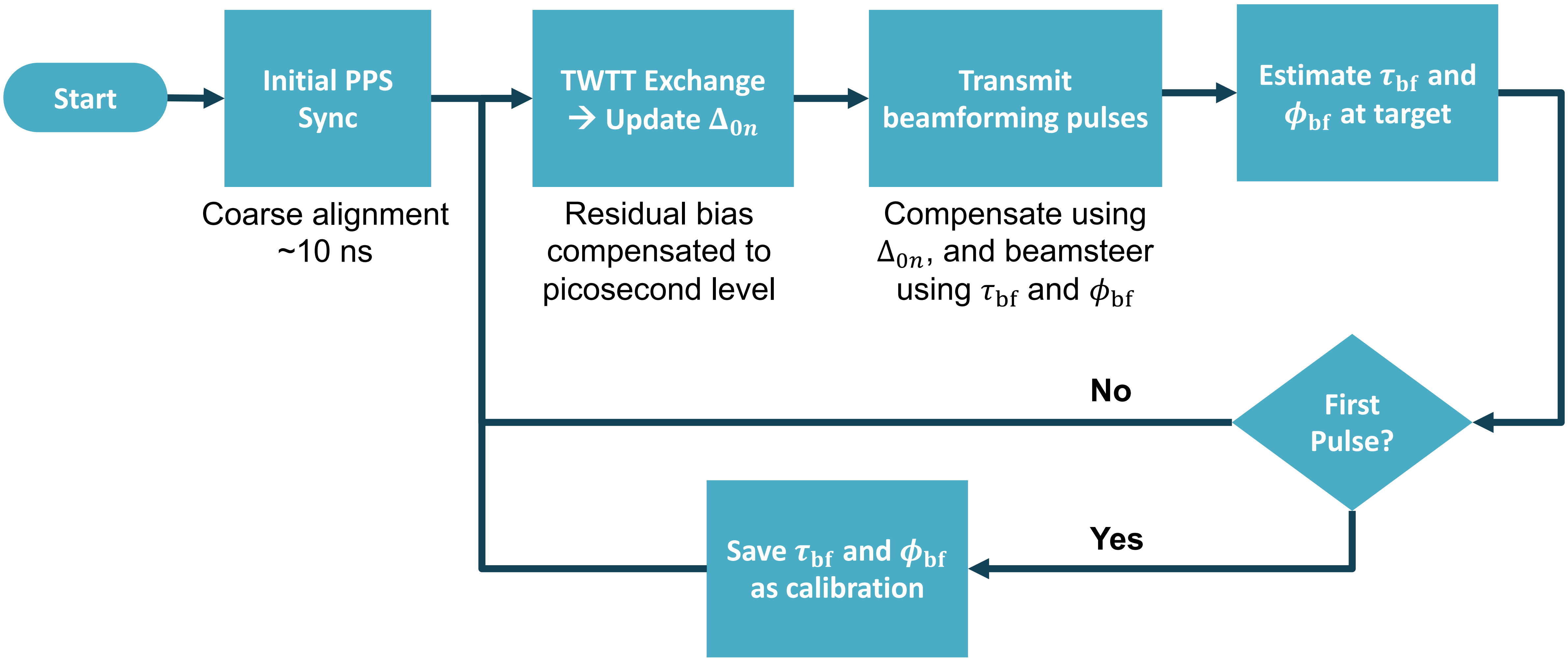
# System Configuration





# System State Flow

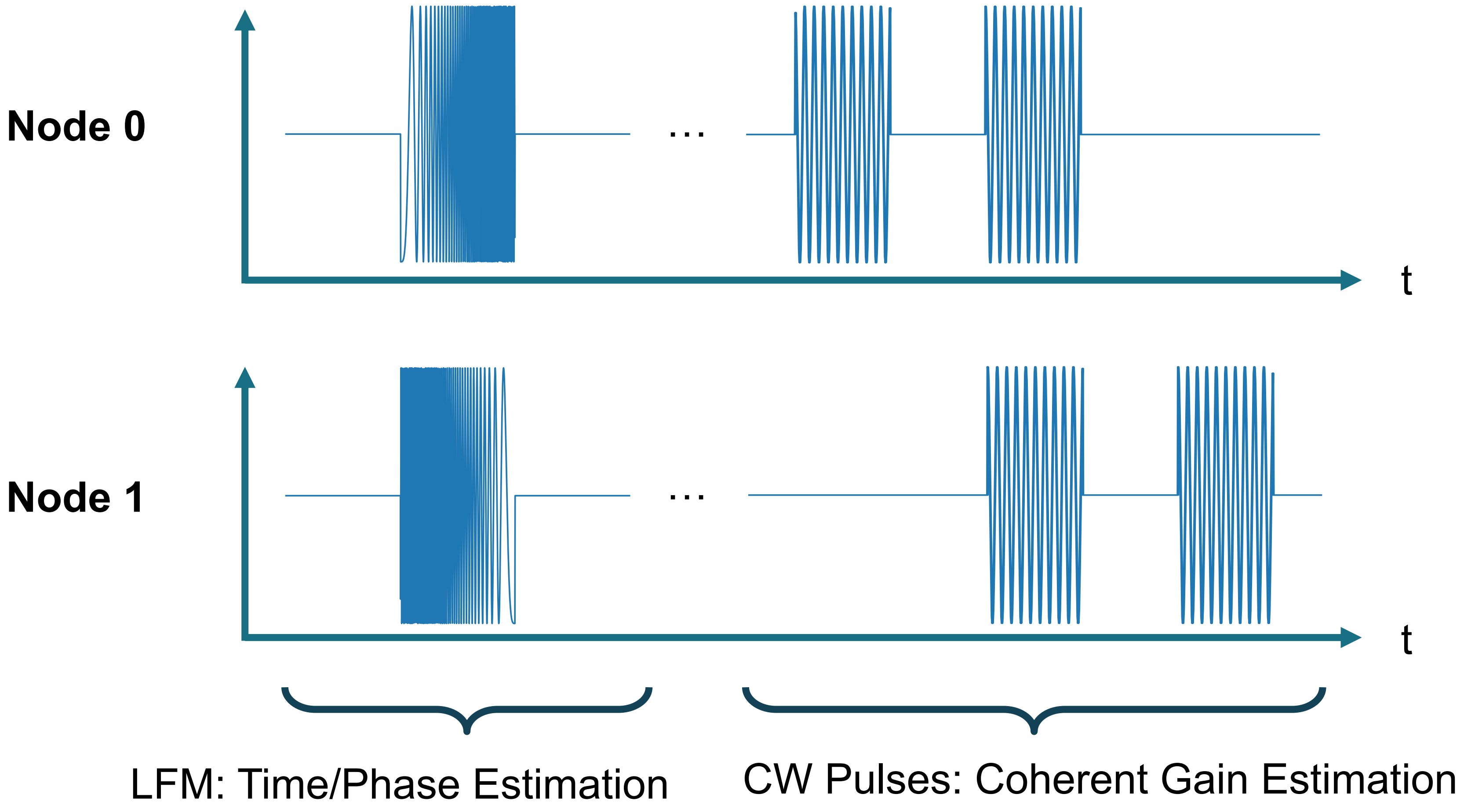




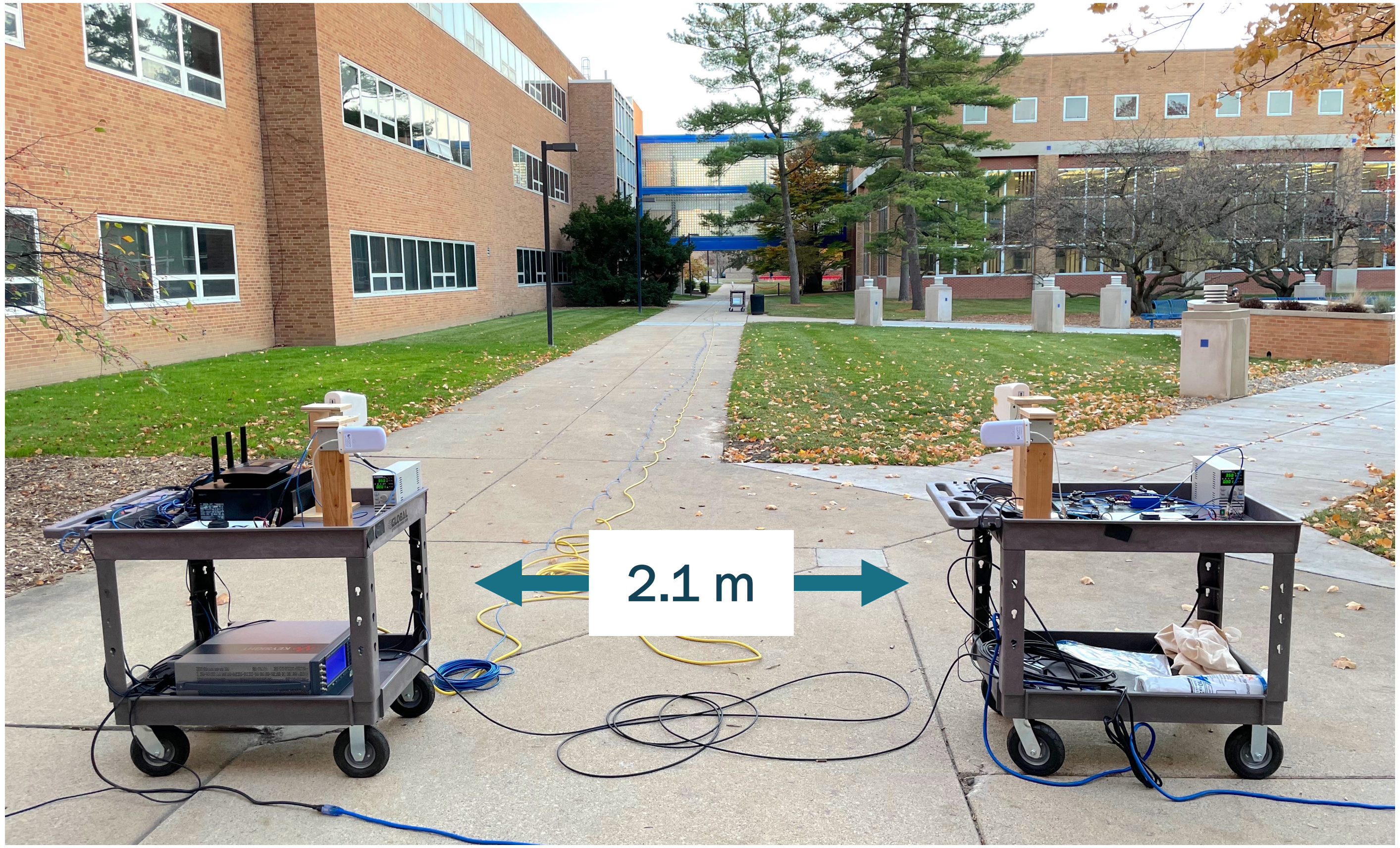
# Beamforming Waveforms



- Each node transmitted orthogonal LFMs followed by two CW pulses



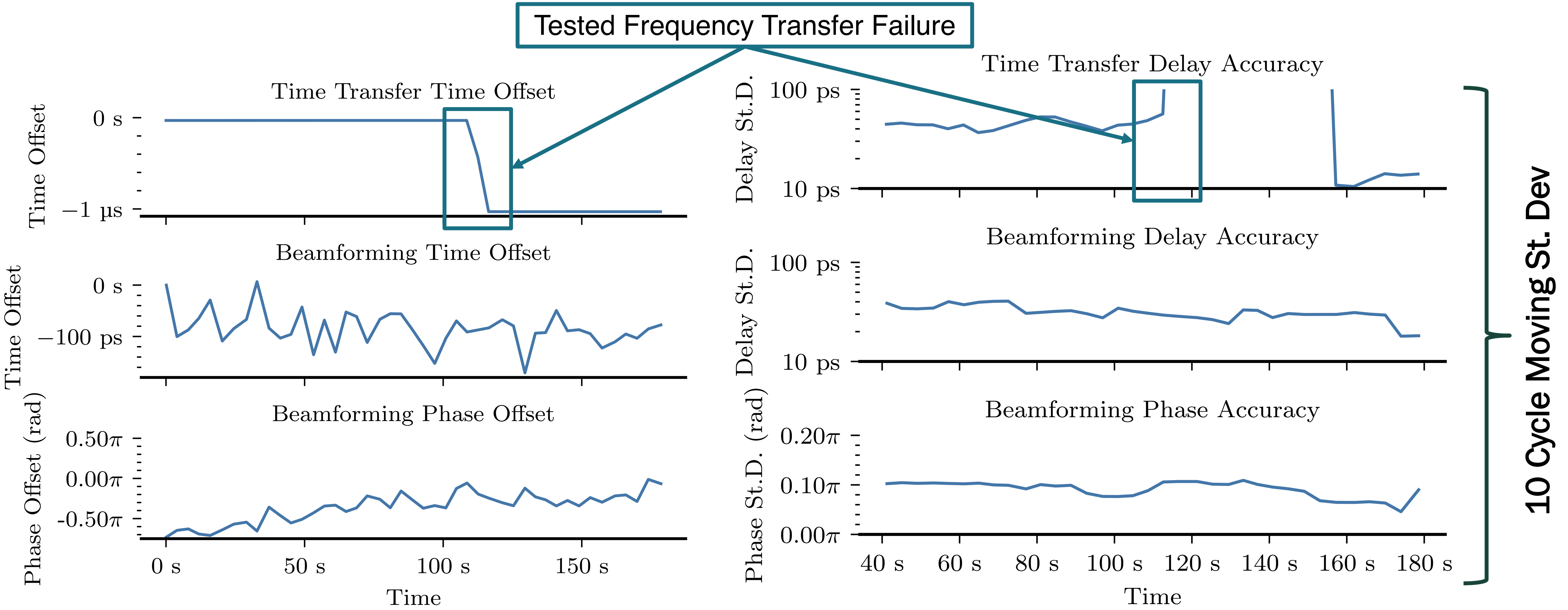
# Beamforming Experiment




Transmit Nodes Setup

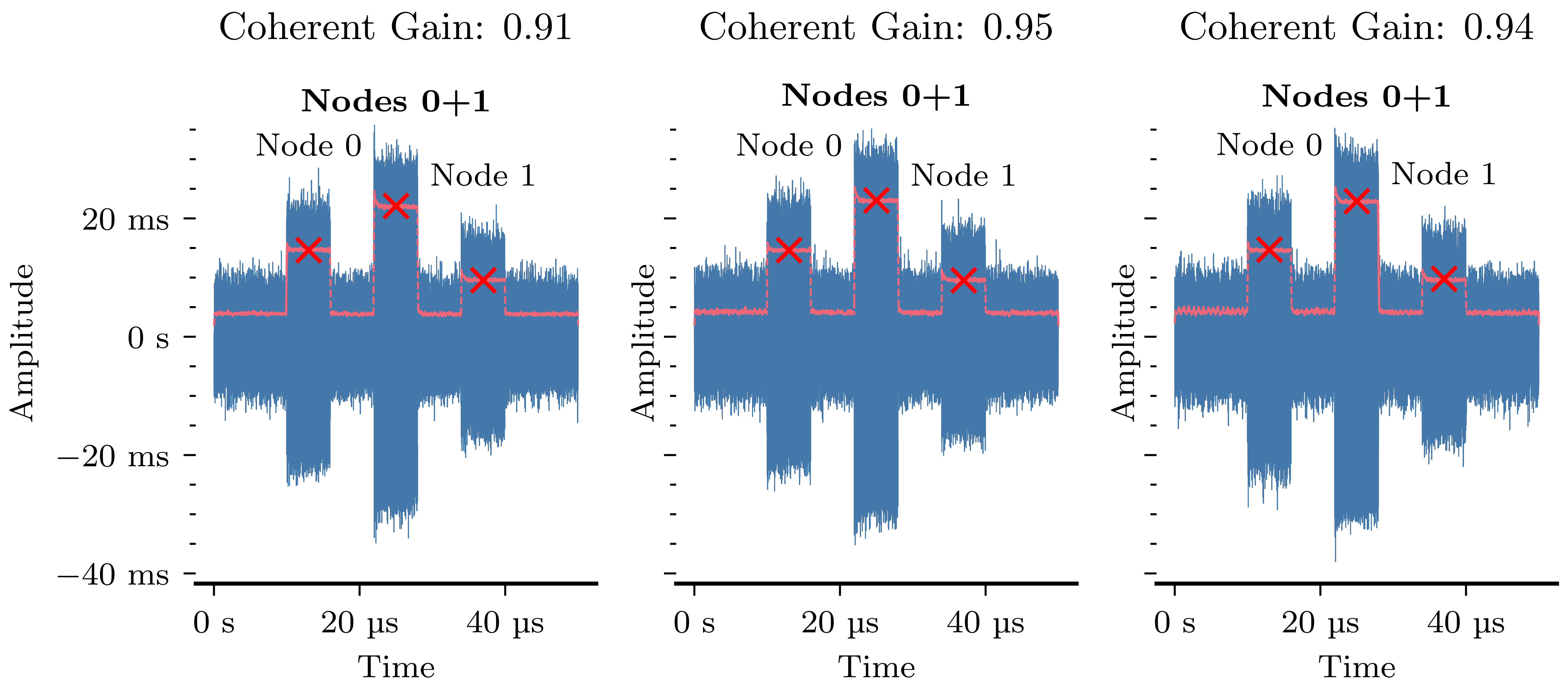
Target Node Setup (41 m downrange)

# Beamforming Experiment




- Time Transfer SNR: ~23dB
- Cycle Time: ~4 s
- Beamforming Std.: $18 < \sigma_{bf} < 40$ ps
- Phase Std.: $0.04\pi < \sigma_{\phi} < 0.10\pi$
- Throughput (BPSK): $2.5 < b_{max} < 5.5$ Gbps
- Max Carrier Freq.: $1.0 < f_{max} < 2.78$ GHz

# Beamforming Experiment



# Conclusion



- Discussed:
  - High-accuracy time transfer technique using spectrally sparse two-tone waveforms
  - Two-step refinement and QLS bias compensation process
- Demonstrated:
  - fully wireless outdoor time-frequency synchronization and beamforming with $G_c > 0.9$ over a 41 m

| Internode Distance | Min. Time Transfer Std. | Min. Beamforming Std. | Max. Throughput (BPSK; $G_c \geq 0.9$) | Max. Carrier Frequency $P(G_c \geq 0.9) \geq 0.9$ |
|---|---|---|---|---|
| 2.1 m | 10.47 ps | 18.00 ps | 5.56 Gbps | 2.78 GHz |
| 5.0 m | 14.79 ps | 24.02 ps | 4.16 Gbps | 2.08 GHz |

# Questions

Thank you to our project sponsors and collaborators:

Lawrence Livermore National Laboratory

Office of Naval Research Science & Technology

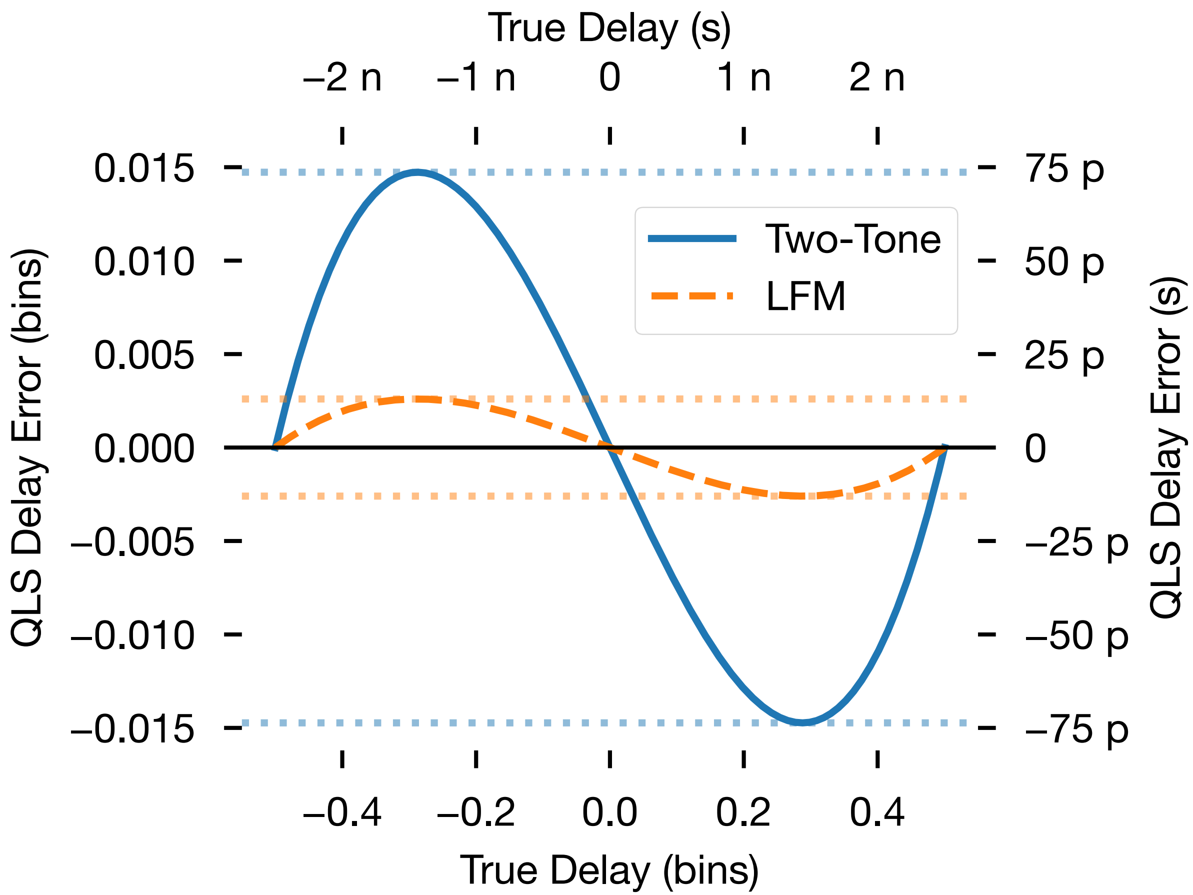
National Science Foundation

# Backup Slides

# Delay Estimation Refinement

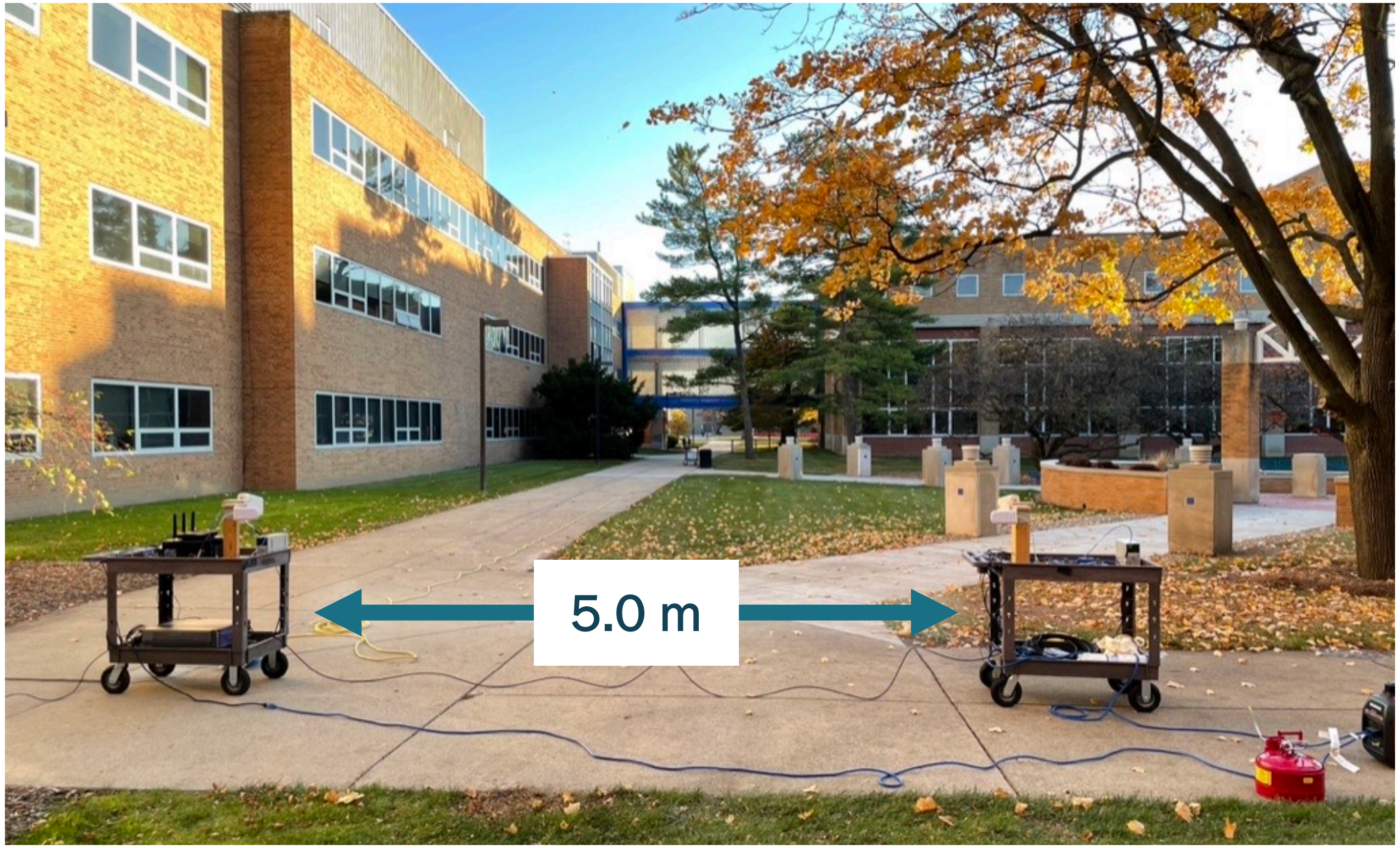


- QLS results in small residual bias due to an imperfect representation of the underlying MF output
- Residual bias is a function of waveform and sample rate
- Can be easily corrected via lookup table



# Baseline 5.0 m



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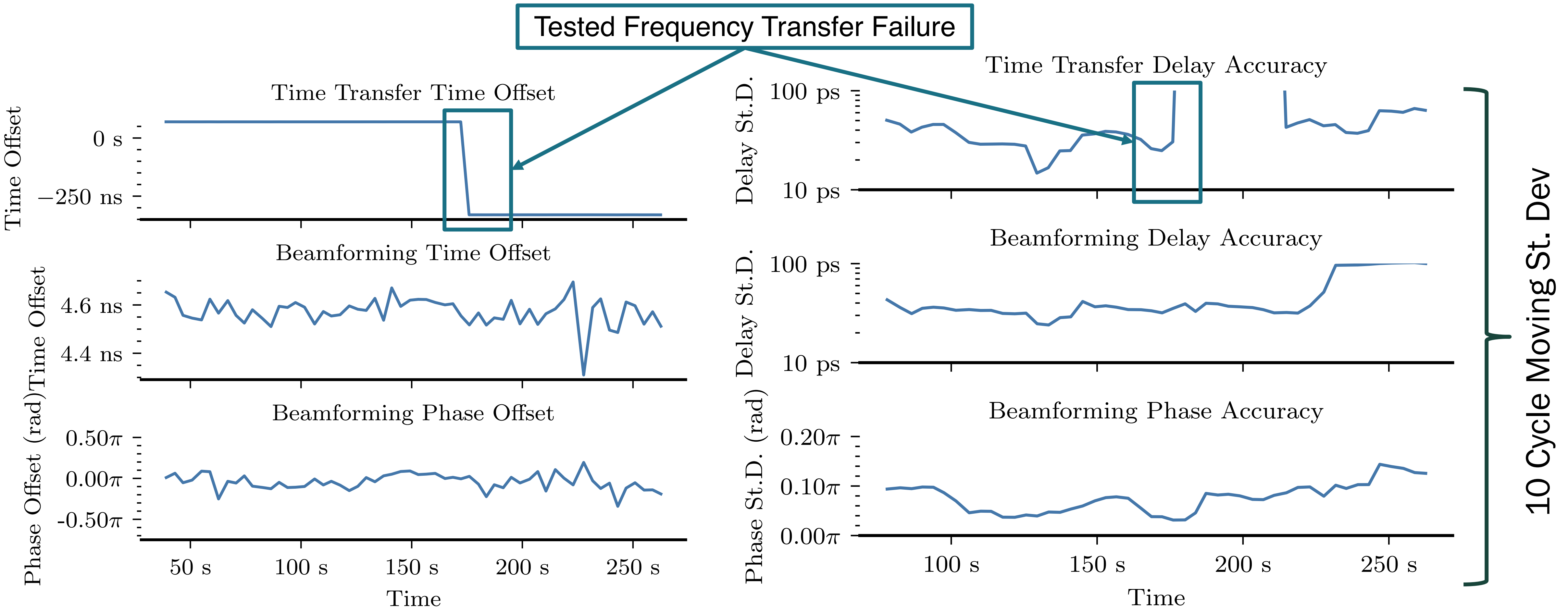
# Baseline 5.0 m






- Time Transfer SNR: ~23dB
- Cycle Time: ~4 s
- Beamforming Std.: $24 < \sigma_{\mathrm{bf}} < 100$ ps
- Phase Std.: $0.05\pi < \sigma_{\phi} < 0.10\pi$
- Throughput (BPSK): $1.0 < b_{\max} < 4.16$ Gbps
- Max Carrier Freq.: $0.67 < f_{\max} < 2.08$ GHz

# Baseline 5.0 m